EE3501 POWER SYSTEM ANALYSIS L T P C 3 0 0 3

UNIT I POWER SYSTEM 9

Need for system planning and operational studies - Power scenario in India - Power system components, Representation - Single line diagram - per unit quantities - p.u. impedance diagram - p.u. reactance diagram, Network graph Theory - Bus incidence matrices, Primitive parameters, Formation of bus admittance matrix – Direct inspection method – Singular Transformation method.

UNIT II POWER FLOW ANALYSIS 9

Bus classification - Formulation of Power Flow problem in polar coordinates - Power flow solution using Gauss Seidel method - Handling of Voltage controlled buses - Power Flow Solution by Newton Raphson method – Flow charts – Comparison of methods.

UNIT III SYMMETRICAL FAULT ANALYSIS 9

Assumptions in short circuit analysis - Symmetrical short circuit analysis using Thevenin's theorem - Bus Impedance matrix building algorithm (without mutual coupling) – Symmetrical fault analysis through bus impedance matrix - Post fault bus voltages - Fault level - Current limiting reactors.

UNIT IV UNSYMMETRICAL FAULT ANALYSIS 9

Symmetrical components - Sequence impedances - Sequence networks - Analysis of unsymmetrical faults at generator terminals: LG, LL and LLG - unsymmetrical fault occurring at any point in a power system.

UNIT V STABILITY ANALYSIS 9

Classification of power system stability – Rotor angle stability - Power-Angle equation – Steady state stability - Swing equation – Solution of swing equation by step by step method – Swing curve, Equal area criterion - Critical clearing angle and time, Multi-machine stability analysis – modified Euler method.

TOTAL : 45 PERIODS

TEXT BOOKS:

1. John J. Grainger, William D. Stevenson, Jr, 'Power System Analysis', Mc Graw Hill Education (India) Private Limited, New Delhi, 2017.

2. Kothari D.P. and Nagrath I.J., 'Power System Engineering', Tata McGraw-Hill Education, 3 rd edition 2019.

3. Hadi Saadat, 'Power System Analysis', Tata McGraw Hill Education Pvt. Ltd., New Delhi, 21st reprint, 2010.

REFERENCES

1. Pai M A, 'Computer Techniques in Power System Analysis', Tata Mc Graw-Hill Publishing Company Ltd., New Delhi, Second Edition, 2007.

2. J. Duncan Glover, Mulukutla S.Sarma, Thomas J. Overbye, 'Power System Analysis & Design', Cengage Learning, Fifth Edition, 2012.

3. P. Venkatesh, B. V. Manikandan, A. Srinivasan, S. Charles Raja, "Electrical Power Systems: Analysis, Security and Deregulation" Prentice Hall India (PHI), second edition - 2017

4. Gupta B.R., 'Power System - Analysis and Design', S. Chand Publishing, Reissue edition 2005.

5. Kundur P., 'Power System Stability and Control', Tata McGraw Hill Education Pvt. Ltd., New Delhi, 2013

UNIT - I POWER SYSTEM

NEED FOR SYSTEM PLANNING AND OPERATIONAL STUDIES:-

Need for power system analysis in planning and operation of power system, operational planning covers the whole period ranging from the incremental stage of system development. The system operational engineers at various points like area, space, regional and national load despatch deals with the despatch of power

power balance equation is

$$P_D = \sum_{i=1}^{N} P_{Gi} \quad , \quad i = 1, 2, 3 \ldots N$$

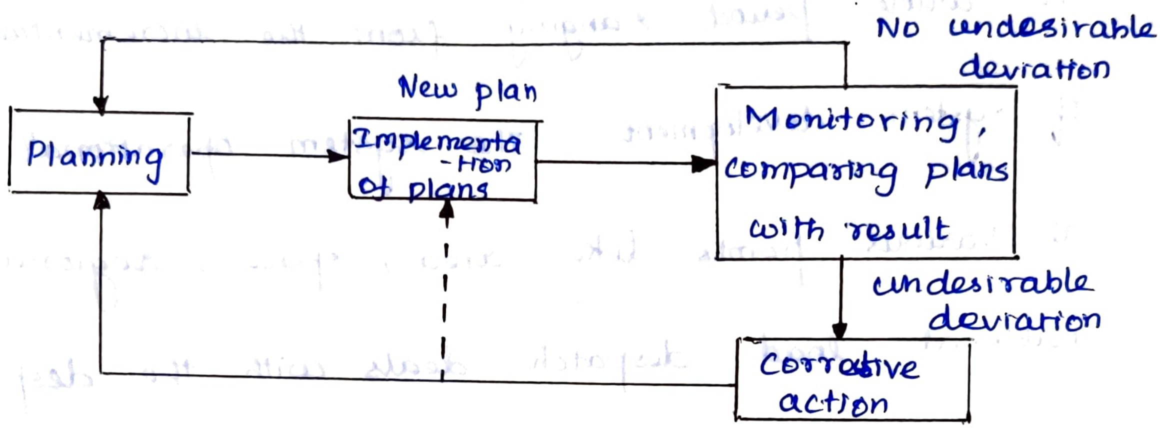
Total demand = Sum of the real power generation.

The operation of a power system must be reliable and uninterrupted. The reliability of power supply implies more than availability of power. The load must be fed at constant voltage and frequency.

Electrical areas are large in size. so planning for future expension of a power system is essential.

Importance of power system planning and operational analysis covers the maintenance of generation, transmission and distribution facilities.

Block diagram for planning and operation of power system:-



Steps to be followed:-

* planning for power system
* Implementation of the plans.
* Monitoring the system
* compare with results.
* If not undesirable deviation occurs, then directly go to the undesirable planning of syste..
* If undesirable deviation occurs, then corrective action and then go to the planning of the system.

For planning and operation of power system, the following analysis are more important

* Load flow analysis
* Short circuit analysis
* Transient analysis.

(i) Load Flow Analysis :-

Normally, electrical power systems operate in their steady state mode and the basic calculation required to determine the characteristics of this state is called as load flow.

Power flow studies are used to determine the voltage, current, active and reactive power flows in a given power system.

The number of operating conditions can be analyzed such as loss of generator, loss of transmission line, loss of transformer or load. These condition may cause equipment overloads or unacceptable voltage levels.

The load flow analysis to determine

* optimum size and location of the capacitors for the power factor improvement.
* starting point for stability analysis.
* planning of new system or the extension of an existing system.
* Evaluate the effect of different loading conditions of an existing system

(2) **Short Circuit Analysis :-**

Short circuit in any part of a power system causes as increase in current and create an abnormal or faulty condition in the system. It performs to determine the magnitude of the current flowing throughout the power system at various time intervals after fault until it reaches a steady state conditions.

The objective of short circuit analysis is to precisely, to determine the currents and voltages at different locations of the system corresponding to different types of faults, such as three phase to ground fault, line to ground fault, line to line fault, double line to ground fault and open conductor fault.

The data is used to select fuses, protective relays and circuit breakers to rescue the system from the abnormal condition. The symmetrical components and sequence networks are used in the analysis of unsymmetrical faults.

(3) Transient stability Analysis:-

Stability may be divided into steady state and transient stability.

Steady state stability

The ability of the power system to remain in synchronism following relatively slow load change or continual changes in generation and switching out of lines.

Transient stability:-

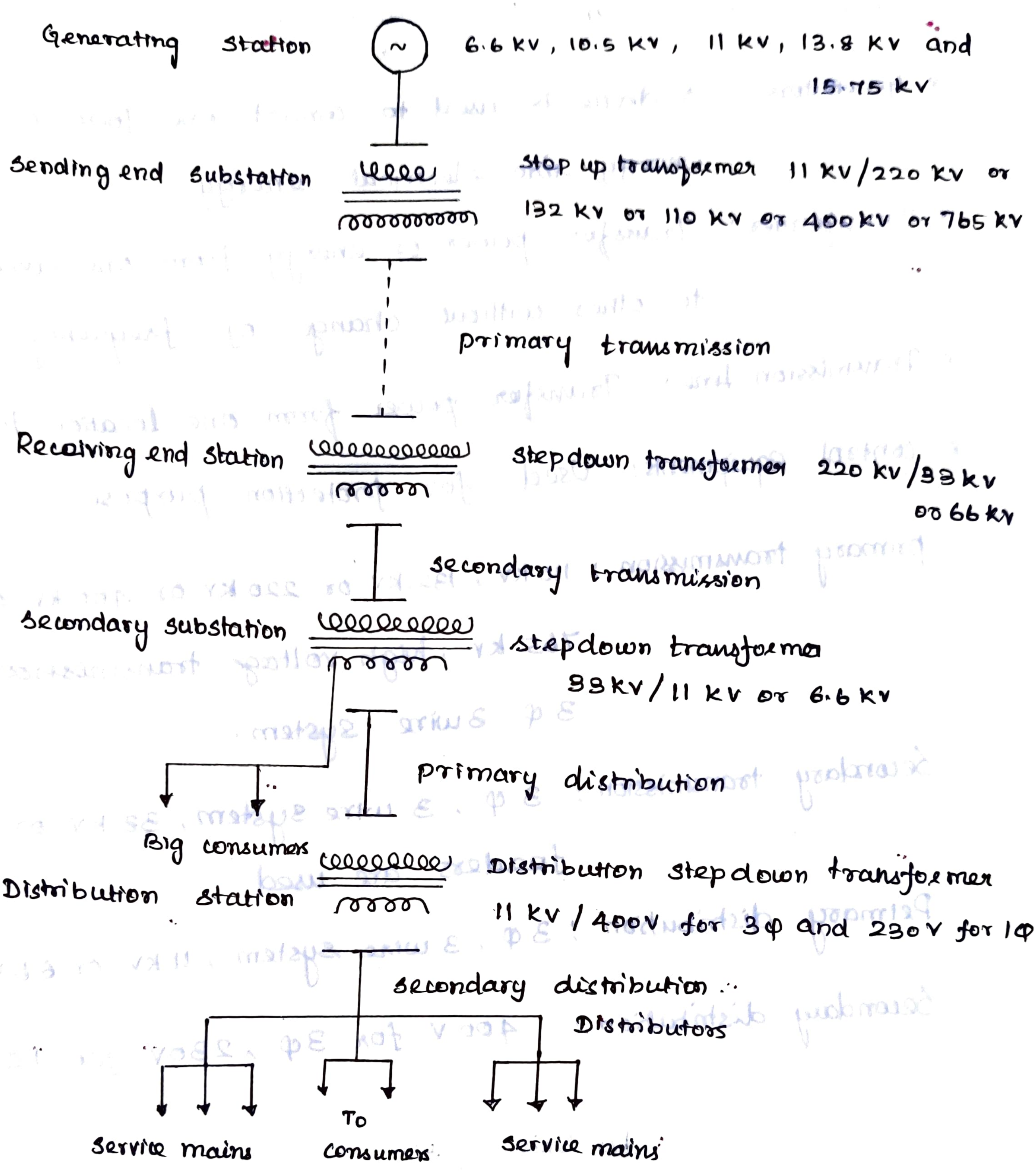
It is defined as ability of the power system to remain synchronism under large disturbance conditions, such as fault and switching operations. The maximum power transfer limit is less than that of the steady state condition.

The transient stability studies are conducted when new generating and transmitting facilities are planned. It uses in determining the nature of relaying system needed, critical clearing time of circuit breakers, voltage level and transfer capability between system, etc.

Power system components:-

(1) structure of power system:-

An electrical power system consists of generation, transmission and distribution. The transmission system supply bulk power and the distribution systems transfer electric power to the ultimate consumers.



Electrical energy is generated in hydro, thermal and nuclear power station. Sometimes, electrical energy is generated from non-renewable energy resources like wind, waves, fossil fuels, etc.

Components of Electric Power Systems :-

* Generators :- A device is used to convert one form of energy into electrical energy.
* Transformer : Transfer power or energy from one circuit to other without change of frequency.
* Transmission Line :- Transfer power from one location to another
* control equipments : Used for protection purpose.

Primary transmission : 110 kV, 132 kV or 220 kV or 400 kV or 765 kV, high voltage transmission, 3 φ 3 wire system.

Secondary transmission : 3 φ, 3 wire system, 33 kV or 66 kV feeders are used.

Primary distribution : 3 φ, 3 wire system, 11 kV or 6.6 kV.

Secondary distribution : 400 V for 3φ, 230V for 1φ.

Per Unit System

Advantages :-

* Per unit data representation yields valuable relative magnitude information.
* Circuit analysis of systems containing transformers of various transformation ratios is greatly simplified.

* Per unit systems are ideal for computerized analysis and simulation of complex power system problems.

* Circuit parameters tends to fall in relatively narrow numerical ranges making erroneous data easy to spot.

* Manufactures usually specify the impedence values of equipment in per unit of the equipments rating. If any data is not available, it is easier to assume its per unit value than its numerical value.

* The ohmic value of impedences as referred to secondary is different from the values as referred to primary. However, if base values are selected properly, the p.u impedence is same as on the two sides of the transformer.

* The circuit laws are valied in p.u systems, power and voltage equations are simplified since the factors of $\sqrt{3}$ and 3 are eliminated.

Define - Per Unit :-

The per unit value of any quantity is defined as the ratio of actual quantity to its base quantity expressed as a decimal. The ~~ratio~~ in percent is 100 times the value in per unit. Both the value have same unit, hence p.u is dimensionless.

Let us assume two base values $|V_b|$ and $|I_b|$ expressed in rms voltage and current respectively.

$$\text{P.U voltage} = \frac{\text{Actual voltage}}{\text{Base voltage}} = \frac{V}{|V_b|}$$

$$\text{P.U current} = \frac{\text{Actual current}}{\text{Base current}} = \frac{I}{|I_b|}$$

$$\text{Base Apparent power} = |S_b| = |V_b|\,|I_b| \text{ VA}$$

$$\text{Base impedence} = |Z_b| = \frac{|V_b|}{|I_b|}\ \Omega$$

$$\text{Apparent power in p.u} = \frac{\text{Actual power}}{\text{Base power}} = \frac{S}{|S_b|}$$

$$= \frac{P + jQ}{|S_b|} = \frac{P}{|S_b|} + \frac{jQ}{|S_b|}$$

$$= P_{p.u} + j\,Q_{p.u}$$

$P_{p.u} \rightarrow$ P.U real power

$Q_{p.u} \rightarrow$ P.U reactive power.

$$\text{Impedence in p.u} = \frac{\text{Actual impedence}}{\text{Base impedence}}$$

$$Z_{p.u} = \frac{Z}{|Z_b|} = \frac{R + jX}{(Z_b)} = \frac{R}{|Z_b)} + \frac{jX}{|Z_b)}$$

$$Z_{p.u} = R_{p.u} + j\,X_{p.u}$$

Circuit Formulas in P.U values (single phase) :-

(1) **Ohm's law,**

$$V_{p.u} = Z_{p.u} \cdot I_{p.u}$$

where base values are always real.

(2) **Impedence at p.u,**

$$\text{Impedence in p.u} = \frac{\text{Actual impedence}}{\text{Base impedence}} = \frac{Z}{|Z_b)}$$

w.k.T. $|S_b| = |V_b)\,|I_b|$

$$|I_b) = \frac{|S_b|}{|V_b)}$$

and $|Z_b) = \dfrac{|V_b|}{|I_b|} = \dfrac{|V_b) \times |V_b)}{|I_b| \times |V_b)} = \dfrac{|V_b)^2}{|S_b|}$

$\therefore$

$$Z_{p.u} = \frac{Z}{|Z_b)} = \frac{Z}{|V_b)^2} \times |S_b)$$

Now V_b in kV. and S_b in MVA

$\therefore$ $\boxed{Z_{p.u} = \dfrac{Z}{|kV_b)^2} \times |MVA_b|}$

and $Z_{p.u} = \dfrac{Z}{|kV_b|^2} \times \dfrac{KVA_b}{1000}$ (S_b in kVA)

Three Phase Circuits :-

Let the three phase volt-ampere S_b or MVA_b. The line to line base voltage V_b or KV_b.

$$\text{Base Current} = \frac{S_b}{\sqrt{3}\, V_b}$$

$$\text{Base impedence} = \frac{V_b}{\sqrt{3}\, I_b}$$

then $$\text{Base impedence} = \frac{V_b}{\sqrt{3} \times S_b} \times \sqrt{3}\, V_b$$

$$\boxed{Z_b = \frac{V_b^2}{S_b} = \frac{KV_b^2}{MVA_b}}$$

Let

$$S_{p.u} = V_{p.u} \times I^*_{p.u}$$

$$V_{p.u} = Z_{p.u} \times I_{p.u}$$

complex load power $S_L^*(p.u) = 3\, V_p^*\, I_p$

$$I_p = \frac{V_p}{Z_p}, \quad Z_p = \frac{V_p}{I_p}$$

and $$Z_p = \frac{|V_p| \times 3\,|V_p|}{S_L^*(3\phi)} = \frac{3\,|V_p|^2}{S_L^*(3\phi)}$$

$$Z_p = \frac{\sqrt{3}\,|V_p| \times \sqrt{3}\,|V_p|}{S_L^*(3\phi)} = \frac{|V_{LL}|^2}{S_L^*(3\phi)}$$

$\therefore$ Base impedence $$Z_p = \frac{|KV_b|^2}{MVA_b} = \frac{|KV_b|^2 \times 1000}{KVA_b}$$

Load impedence. $Z_b = \frac{Z_p \text{ (actual)}}{Z \text{ base}}$

$$= \frac{|V_{LL}|^2}{S_L^*(3\phi)} \times \frac{S_b}{|V_b|^2} = \frac{|V_{LL}|^2}{|V_b|^2} \times \frac{S_b}{S_L^*(3\phi)}$$

$$\therefore \boxed{Z_{pu} = \frac{|V_{p.u}|^2}{S_{L(pu)}^*} = \frac{|V_{p.u}|^2}{P - jQ}}$$

Change of Base :-

The actual value of impedence depends only on the materials and construction, is unchanged by a change in the rating of the machine. However, if the base is changed, the per unit impedence of the machine takes on a new value,

$$\text{Per unit impedence} = \frac{\text{Actual impedence}}{\text{Base impedence}}$$

$$= \frac{\text{Actual impedence}}{\text{Base kV}^2} \times \text{Base MVA}$$

To change from per unit impedence on a given base to per unit impedence on a new ~~phase~~ base

$$Z_{p.u} \text{ given} = \frac{Z_{actual}}{kV_b^2 \text{ given}} \times MVA_b \text{ given}$$

$$Z_{actual} = Z_{p.u} \text{ given} \times \frac{kV_b^2 \text{ given}}{MVA_b \text{ given}}$$

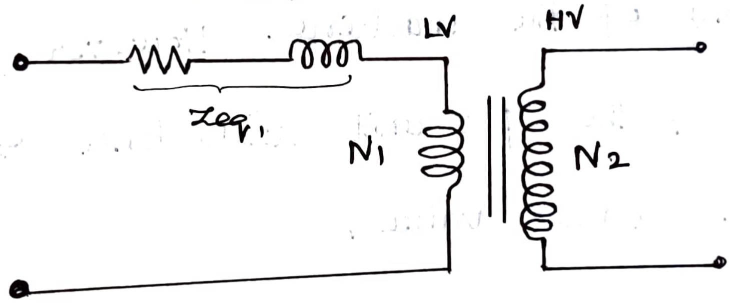
$$Z_{p.u\ new} = \frac{Z_{actual}}{KV_{b\ new}^2} \times MVA_{b\ new}.$$

$$\therefore Z_{p.u\ new} = Z_{p.u\ given} \times \left[\frac{KV_{b\ given}}{KV_{b\ new}}\right]^2 \times \left[\frac{MVA_{b\ new}}{MVA_{b\ given}}\right]$$

Per Unit Impedence of two winding transformer :-

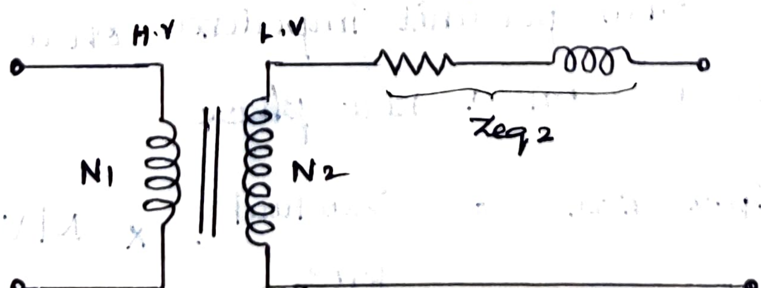
(a) The approximate equivalent circuit of two winding transformer with impedence referred to low voltage side (primary) :-

The equivalent circuit of single phase transformer referred to L.V side,



$$Z_{p.u\ (primary)} = Z_{eq_1} \times \frac{|MVA_b|}{|KV_b|^2}$$

(b) The approximate equivalent circuit with impedence referred to high voltage side (secondary)



$$Z_{p.u\ (secondary)} = Z_{eq_2} \times \frac{MVA_b}{|KVA_b|^2} \quad ---(1)$$

and, $\frac{N_2^2}{N_1^2} = \frac{Z_{eq2}}{Z_{eq1}} \Rightarrow Z_{eq2} = Z_{eq1} \times \frac{N_2^2}{N_1^2}$

since $N \propto V$

$$\therefore \quad Z_{eq2} = Z_{eq1} \times \frac{|KV_{b2}|^2}{|KV_{b1}|^2} \quad \text{- - - - (2)}$$

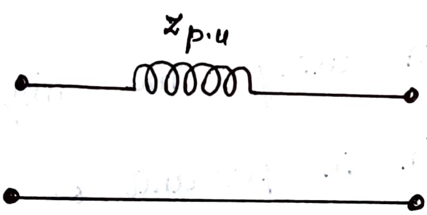
sub eqn ② in eqn ①; we get

$$Z_{p.u}\,(\text{secondary}) = Z_{eq1} \frac{|KV_{b2}|^2}{|KV_{b1}|^2} \times \frac{MVA_b}{|KV_{b2}|^2}$$

$$\therefore Z_{p.u}\,(\text{secondary}) = Z_{eq1} \frac{MVA_b}{|KV_{b1}|^2}$$

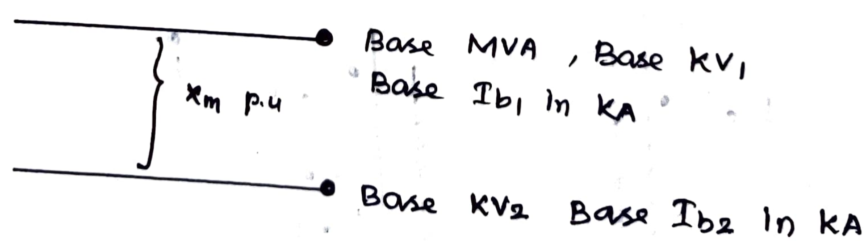
$$\therefore Z_{pu}\,(\text{secondary}) = Z_{p.u}\,(\text{primary}) = Z_{p.u}.$$

Equivalent representation of two winding transformer expressed in p.u



Mutual Inductance in P.U Between Lines of Different voltage levels:-

Let us consider two three phase lines of different voltage levels, with mutual reactance X_m,



Referred to line 2,

$$\text{Per unit mutual impedence} = \frac{\text{Actual Mutual impedence}}{\text{Base mutual impedence}}$$

$$= \frac{X_m}{X_{mb}} = \frac{X_m}{|KV_{b2}|} \times |I_b|$$

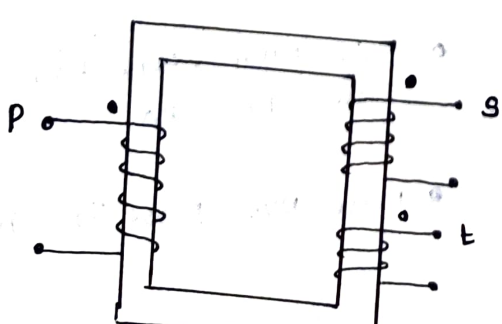
For simplifying,

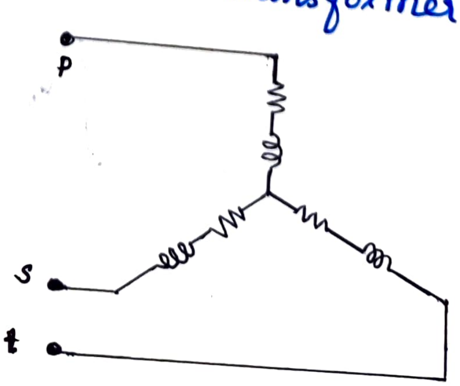
$$\text{Per unit mutual impedence} = \frac{X_m}{|KV_{b2}|} \times |I_{b1}| \times \frac{|KV_{b1}|}{|KV_{b1}|}$$

$$X_{p.u} = \frac{X_m \, |MVA_{b1}|}{|KV_{b1}| \, |KV_{b2}|}$$

Three Winding Transformer :-

In three winding transformer, all the three windings have different MVA rating. The impedence of each winding may given in per unit or percent based on its own MVA rating. But all per unit impedences are expressed on same MVA base. Base kV is taken differently for three windings that depends on line voltages of three circuit of the transformer.





Let Z_{ps} be the leakage impedence measured in primary with secondary short circuited and teritiary open.

$$Z_{ps} = Z_p + Z_s \quad \text{——— ①}$$

Let Z_{pt} be the leakage impedence measured in primary with teritiary short circuited and secondary open.

$$Z_{pt} = Z_p + Z_t \quad \text{——— ②}$$

Let Z_{st} be the leakage impedence measured in secondary with tertiary short circuited and primary open.

$$Z_{st} = Z_s + Z_t \quad \text{——— ③}$$

Determine Z_p, Z_s and Z_t using Z_{ps}, Z_{pt} & Z_{st}

Equation ① + ② − ③, we get

$$Z_{ps} + Z_{pt} - Z_{st} = 2Z_p + Z_s + Z_t - Z_s - Z_t$$

$$Z_p = \frac{1}{2}\left[Z_{ps} + Z_{pt} - Z_{st}\right] \quad \text{——— ④}$$

Similarly,

$$Z_t = \frac{1}{2}\left[Z_{pt} + Z_{st} - Z_{ps}\right] \quad \text{——— ⑤}$$

$$Z_s = \frac{1}{2}\left[Z_{ps} + Z_{st} - Z_{pt}\right] \quad \text{——— ⑥}$$

Mostly the primary and secondary windings are star connected and the tertiary winding is delta connected for three phase operation.

Selection of Base Values:-

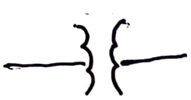
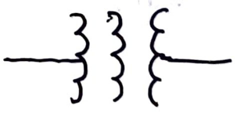
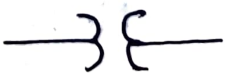
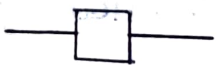
(i) Selection of Base MVA.

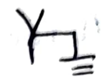
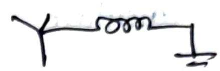
First a base MVA is chosen for the network. The Same MVA will be used in all parts of the system. It may be the largest MVA of a section, or total MVA of the System or any value like 10, 100, 1000 MVA etc.

(ii) Selection of Base kV:-

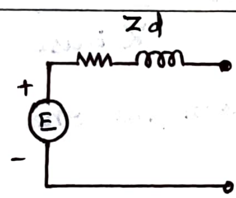
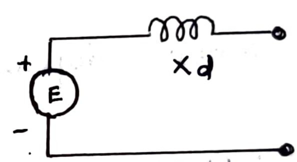
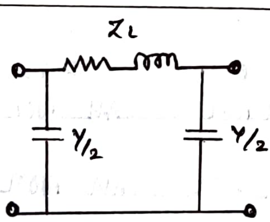
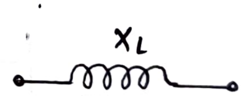
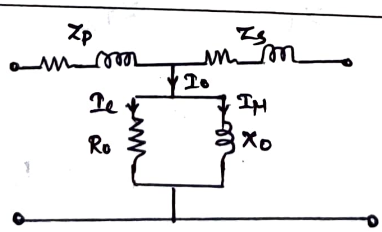
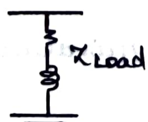
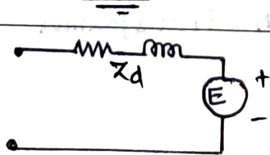
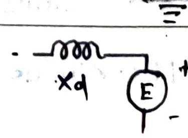
The rated voltage of the largest section may be taken as base kV. The base voltages of remaining sections are assigned depends on the turns ratio of the transformer.

Single Line Diagram (or) one line Diagram:-

Alternator or synchronous Motor	
Two winding power transformer	
Three winding power Transformer	
current Transformer	
Potential Transformer	 or 
Transmission line	
Power Circuit breaker (oil or liquid)	

Air blast circuit breaker	
3 φ, 3 wire delta connection	
3 φ, star connection, neutral ungrounded	
3 φ, Y connection, neutral solidly grounded	
3 φ, Y connection, neutral solidly grounded through resistor	
3 φ, Y connection, neutral solidly grounded through reactor	

Perphase Representation of components of power system :-

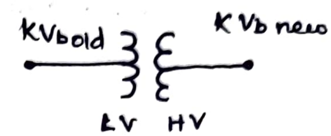
Component	Equivalent Circuit	Reactance diagram
Alternator :		
Transmission Line		
Transformer		
Load		
Synchronous Motor		

STEPS TO DRAW PER UNIT IMPEDENCE DIAGRAM :-

* choose common MVA or base MVA for the system (Mostly highest generator rating is taken).

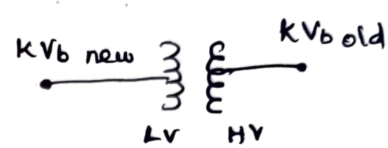
* choose an approximate base KV for each and every section.

$$KV_{b\,new} = KV_{b\,old} \times \frac{HT \text{ side rating}}{LT \text{ side rating}}$$



or

$$KV_{b\,new} = KV_{b\,old} \times \frac{LT \text{ side rating}}{HT \text{ side rating}}$$



* Calculate per unit impedence at each section,

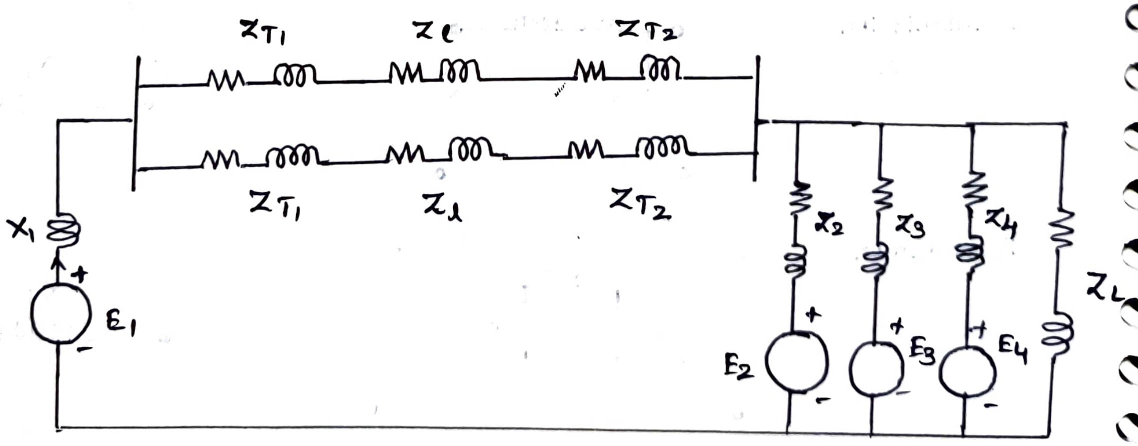
For generator, transformer, motor

$$Z_{p.u\,new} = Z_{p.u\,given} \times \left[\frac{KV_{b\,given}}{KV_{b\,new}}\right]^2 \times \left[\frac{MVA_{b\,new}}{MVA_{b\,given}}\right]$$

For transmission line, $Z_{p.u} = \frac{Z_{actual}}{Z_{base}} = \frac{Z_{actual}}{(KV_b)^2} \times MVA_b$

* Draw impedence diagram from the one line diagram.

IMPEDENCE DIAGRAM :-

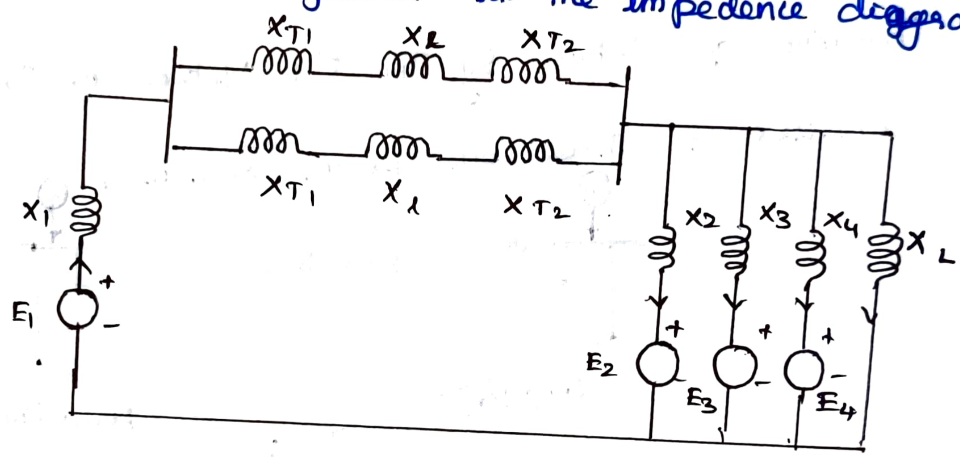


* Single phase transformer equivalent circuit are shown as ideal transformer with transformer impedence indicated on approximate side.

* Magnetization reactances of the transformers have been neglected.
* Generators are represented as voltage sources with series reactance (resistance) and inductive reactance.
* The shunt capacitance are also neglected.
* Loads are represented by resistance and inductive reactance
* Neutral grounding impedences are neglected.

REACTANCE DIAGRAM :-

* All the resistance are neglected in the impedence diagram.



BUS ADMITTANCE MATRIX

Bus Admittance Matrix or Y-bus matrix finds application in load flow and optimal load flow analysis as well as stability analysis. Z-bus matrix or Bus impedence matrix finds application in short circuit analysis.

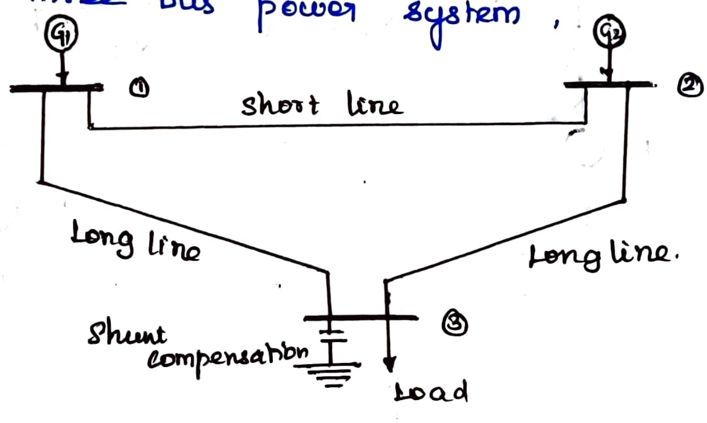
Under steady state condition,

$$[Y]\,[V] = [I]$$

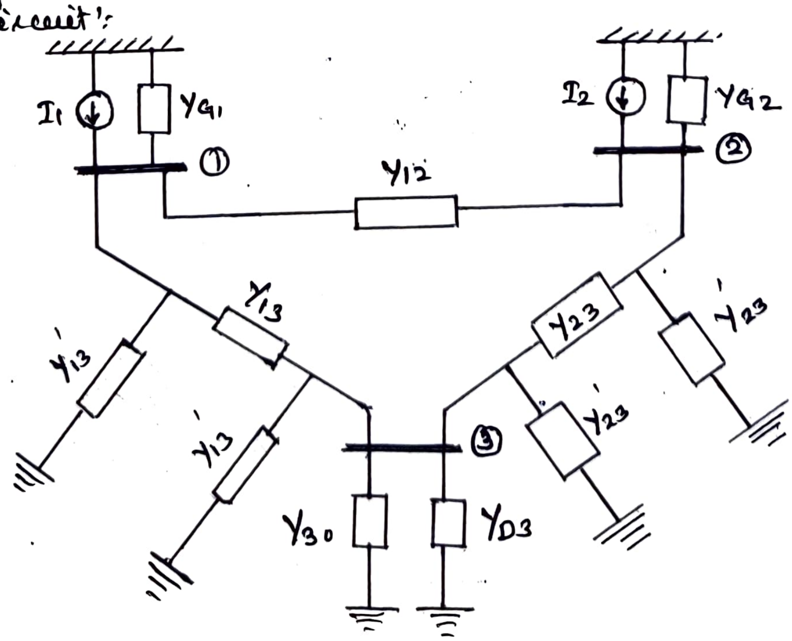
$$[Z]\,[I] = [V]$$

Formation of Y-bus by Two Rule Method or Inspection Method

consider a Three bus power system,



Equivalent Circuit:-



Y — admittance

Y' — Half line admittance charging

Y_D → admittance represented for load

In equivalent circuit,

* Generators are replaced by Norton's equivalent
* Load is replaced by equivalent admittances
* Lines are replaced by π-equivalent network
* Admittance of generators, loads and transmission lines are given in per unit.
* Ground is taken as reference node.

Apply Kirchoff's current law to nodes 1, 2 and 3

Node 1 :-

$$Y_{G1} V_1 + Y_{12}[V_1 - V_2] + Y_{13}[V_1 - V_3] + Y_{13}' V_1 = I_1$$

$$V_1 [Y_{G1} + Y_{12} + Y_{13} + Y_{13}'] - V_2 Y_{12} - V_3 Y_{13} = I_1 \quad \text{--- (1)}$$

Node 2 :-

$$Y_{G2} V_2 + Y_{12}[V_2 - V_1] + Y_{23}[V_2 - V_3] + Y_{23}' V_2 = I_2$$

$$-V_1 Y_{12} + V_2 [Y_{G2} + Y_{12} + Y_{23} + Y_{23}'] - V_3 Y_{23} = I_2 \quad \text{--- (2)}$$

Node 3 : -

$$Y_{D3} V_3 + Y_{30} V_3 + Y_{13}[V_3 - V_1] + Y_{13}' V_3 + Y_{23}[V_3 - V_2] + Y_{23}' V_3 = 0$$

$$-V_1 Y_{13} - V_2 Y_{23} + V_3 [Y_{D3} + Y_{30} + Y_{13} + Y_{13}' + Y_{23} + Y_{23}'] = 0 \quad \text{--- (3)}$$

$$\begin{bmatrix} Y_{G1}+Y_{12}+Y_{13}+Y_{13}' & -Y_{12} & -Y_{13} \\ -Y_{12} & Y_{G2}+Y_{12}+Y_{23}+Y_{23}' & -Y_{23} \\ -Y_{13} & -Y_{23} & Y_{D3}+Y_{30}+Y_{13}+Y_{13}'+Y_{23}+Y_{23}' \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ 0 \end{bmatrix}$$

In general,

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ 0 \end{bmatrix}$$

where Y_{ii} is equal to the sum of the admittance of all elements connected to the i^{th} node.

Y_{ij} is equal to the negative of the sum of the admittances of all elements connected between the node i and j.

$Y_{ij} = 0$ if there is no line between the buses i and j

The current equation is,

$$I_i = \sum_{j=1}^{N} Y_{ij} V_j \quad , \quad i = 1, 2, \dots N$$

$$Y_{ij} (i \neq j) = \frac{I_i}{V_j} \quad (\text{all } V=0 \text{ except } V_j)$$

$=$ Short circuit transfer admittance

$$Y_{ii} = \frac{I_i}{V_i} \quad (\text{all } V = 0 \text{ except } V_i)$$

= short circuit driving point

Important points:-

* Y-bus is $n \times n$ matrix where n is the number of buses.
* The diagonal elements of Y-bus are the driving point admittances and the off-diagonal elements of Y-bus are the short circuit transfer admittances.
* Y_{ij} $(i \neq j) = 0$ if i^{th} and j^{th} buses are not connected.
* Y-bus matrix is symmetric matrix $(Y_{ij} = Y_{ji})$ if the regulating transformers are not involved. so only $\frac{n \times n - n}{2} + n = \frac{n(n+1)}{2}$ terms to be stored for n-bus system.
* Bus admittance matrix is symmetric along the leading diagonal, and we need to store the upper triangular admittance matrix only.
* Each bus is connected to only a few nearby buses, so many off diagonal elements are zero. such matrix is called 'sparse'.

Applications :-

* Y - bus is used in solving load flow Probleme.
* It has gained applications owing to the simplicity in data preparation.
* It can be easily formed and modified for any changes in the network.
* It reduces computer memory and time requirements because of sparse matrix.

Addition of line :-

$$Y_{ii}\text{ new} = Y_{ii}\text{ old} + y$$
$$Y_{ij}\text{ new} = Y_{ij}\text{ old} - y$$
$$Y_{ji}\text{ new} = Y_{ji}\text{ old} - y$$
$$Y_{jj}\text{ new} = Y_{jj}\text{ old} + y$$

Removal of a line :-

$$Y_{ii}\text{ new} = Y_{ii}\text{ old} - y$$
$$Y_{ij}\text{ new} = Y_{ij}\text{ old} + y$$
$$Y_{ji}\text{ new} = Y_{ji}\text{ old} + y$$
$$Y_{jj}\text{ new} = Y_{jj}\text{ old} - y$$

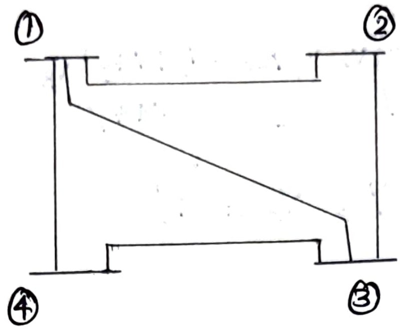
Addition of Shunt Element :-

$$Y_{ii}\text{ new} = Y_{ii}\text{ old} + y$$

(Addition of an element of admittance y from bus 'i' to ground will only affect Y_{ii}).

Elimination of a node or Bus [Gaussian Elimination or Kron Reduction Method].

To minimize computational effort and computer storage, successive elimination or Gaussian elimination method is applicable.



Nodal equations are,

$$Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4 = I_1 \quad \text{——— } ①$$

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 = I_2 \quad \text{——— } ②$$

$$Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4 = I_3 \quad \text{——— } ③$$

$$Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4 = I_4 \quad \text{——— } ④$$

To eliminate node 4, the following steps to be taken

Step 1: divide eqn ④ by Y_{44}

$$\frac{Y_{41}}{Y_{44}}V_1 + \frac{Y_{42}}{Y_{44}}V_2 + \frac{Y_{43}}{Y_{44}}V_3 + \frac{Y_{44}}{Y_{44}}V_4 = \frac{I_4}{Y_{44}} \quad \text{—— } ⑤$$

Step 2: Multiplying eqn ⑤ by Y_{14}

$$\frac{Y_{14}Y_{41}}{Y_{44}}V_1 + \frac{Y_{14}Y_{42}}{Y_{44}}V_2 + \frac{Y_{14}Y_{43}}{Y_{44}}V_3 + \frac{Y_{14}Y_{44}}{Y_{44}}V_4 = \frac{Y_{14}}{Y_{44}}I_4 \quad \text{—— } ⑥$$

Step 3:- Subtract from eqn ①, we get

$$\left(Y_{11} - \frac{Y_{14}Y_{41}}{Y_{44}}\right)V_1 + \left(Y_{12} - \frac{Y_{14}Y_{42}}{Y_{44}}\right)V_2 + \left(Y_{13} - \frac{Y_{14}Y_{43}}{Y_{44}}\right)V_3 + \overbrace{\left(Y_{14} - \frac{Y_{14}Y_{44}}{Y_{44}}\right)}^{\nearrow 0}V_4 = I_1 - \frac{Y_{14}}{Y_{44}} \times I_4$$

We can write the above eqn,

$$Y_{11}' V_1 + Y_{12}' V_2 + Y_{13}' V_3 + Y_{14}' V_4 = I_4'$$

. Similarly multiply eqn ⑤ by Y_{24}, Y_{34} and subtract from eqn ② & ③, we get,

$$\left(Y_{21} - \frac{Y_{24} Y_{41}}{Y_{44}}\right) V_1 + \left(Y_{22} - \frac{Y_{24} Y_{42}}{Y_{44}}\right) V_2 + \left(Y_{23} - \frac{Y_{24} Y_{43}}{Y_{44}}\right) V_3 = I_2 - \frac{Y_{24}}{Y_{44}} I_4$$

$$Y_{21}' V_1 + Y_{22}' V_2 + Y_{23}' V_3 = I_2'$$

and

$$\left(Y_{31} - \frac{Y_{34} Y_{41}}{Y_{44}}\right) V_1 + \left(Y_{32} - \frac{Y_{34} Y_{42}}{Y_{44}}\right) V_2 + \left(Y_{33} - \frac{Y_{34} Y_{43}}{Y_{44}}\right) V_3 = I_3 - \frac{Y_{34}}{Y_{44}} I_4$$

$$Y_{31}' V_1 + Y_{32}' V_2 + Y_{33}' V_3 = I_3'$$

In general, $Y_{ij}' = Y_{ij} - \dfrac{Y_{in} Y_{nj}}{Y_{nn}}$

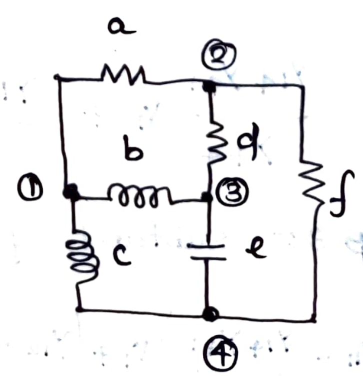
$$Y_{ij\,new} = Y_{ij\,old} - \frac{Y_{in} Y_{nj}}{Y_{nn}}, \quad i = 1, 2, 3 \ldots n, \; i \neq \ldots \quad j = 1, 2, 3 \ldots n, \; j \neq \ldots$$

where 'n' is the node which is to be removed.

Formation of Y-Bus By singular Transformation:

Graph Theory:-

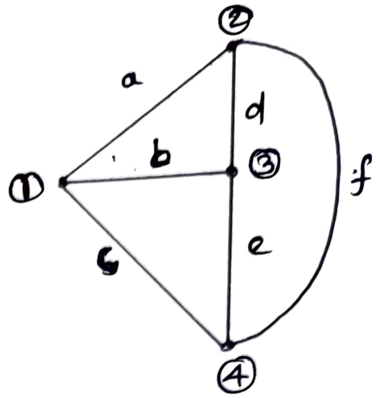
(1) Network:-



Network is an interconnection of elements in various branches at different nodes.

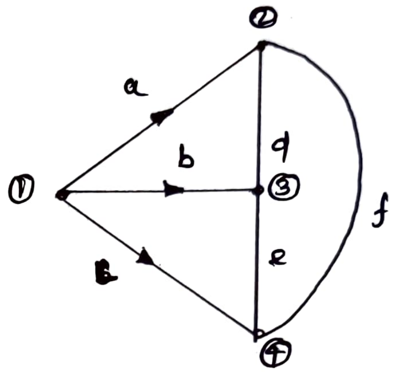
Graph :-

A Graph is representation of network obtained by replacing every element of network by a line segment and every junction point by a node.



Oriented Graph:-

If every branch of a graph has direction, then the graph is called as a directed graph or oriented graph.



Branch or edge:-

A branch is represented by a line segment in the graph of a network.

Node or Bus or Vertices:-

A node is a terminal of a branch which is represented by apoint.

Loop or Closed path:-

If a starting node and ending node is same for a path, then it is called as closed path or loop.

Tree or Twig:-

A tree is a Subgraph of a network which consists of all the nodes as in the graph but has no closed paths.

Properties of Trees:-

* Number of nodes in a graph = Number of nodes in the tree of that graph.
* All the nodes must be connected by elements called tree branches.
* Tree branches must not form any loop or closed path in the sub graph.
* Every connected graph atleast one tree.
* Rank of tree = Rank of graph.
* Number of tree branches = Number of nodes - one [n-1]

Link or Chord:-

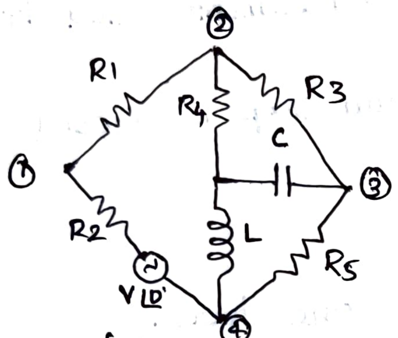
The removal branches of the tree is called links. The branches of cotree is called link or chord. The set of all links of given tree is called the cotree of the graph.

CUT SET SCHEDULE :-

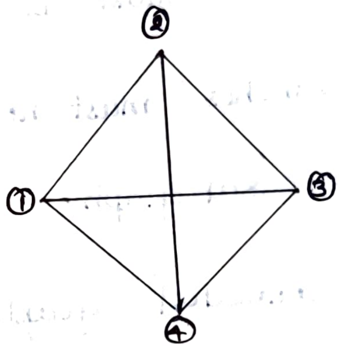
An alternative method of finding out branch voltages is called cut set schedule.

Cutset :- It consists of one and only one branch of the network tree, together with any links which must be cut to divide the network into parts.

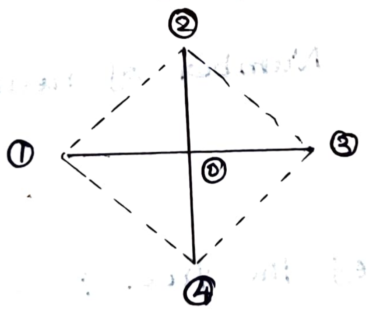
Example



(Network)



Step 1 :- Draw a tree

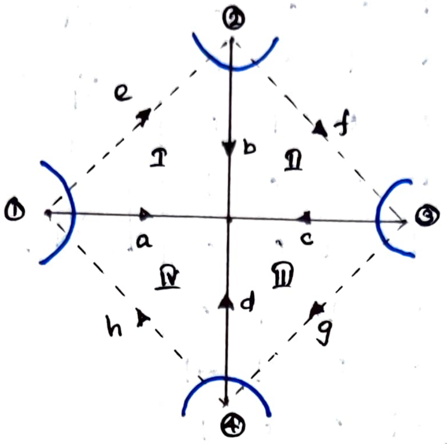


Step 2 :- Give naming for the tree branches first and then to the link.

Assume tree branch direction away from the cut set assume link direction as clockwise.

Step 3 :- Form bus incident matrix

(a) Write the matrix equation for each cut-set using KVL



Let $I_{ba}, I_{bb}, I_{bc}, I_{bd}, I_{be}, I_{bf}, I_{bg}, I_{bh}$ be the branch currents for the elements a, b, c, d, e, f, g, h

$$\begin{array}{c} \\ \text{cut-set I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} \begin{array}{c} \begin{array}{cccc:cccc} a & b & c & d & e & f & g & h \end{array} \\ \left[\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right] \end{array} \begin{bmatrix} I_{ba} \\ I_{bb} \\ I_{bc} \\ I_{bd} \\ I_{be} \\ I_{bf} \\ I_{bg} \\ I_{bh} \end{bmatrix} = 0$$

$[A_b : A_l]\,[I_b] = 0$, $[A] \rightarrow$ Bus incidence matrix

$[I_b] \rightarrow$ Branch current matrix

Number of cut-set = Number of tree branches = 4

(b) Express Branch voltages in terms of Tree Branch voltages

Let V_{bx} be the branch voltages, V_x be the tree branch voltages.

Branch Voltages are $V_{ba} = V_a$, $V_{bb} = V_b$, $V_{bc} = V_c$ & $V_{bd} = V_d$

Loop equations 1 : $V_b - V_a + V_{be} = 0 \Rightarrow V_{be} = V_a - V_b$

Loop equations 2 : $V_{bf} + V_c - V_b = 0 \Rightarrow V_{bf} = V_b - V_c$

Loop equations 3 : $V_{bg} + V_d - V_c = 0 \Rightarrow V_{bg} = V_c - V_d$

Loop equations 4 : $V_{bh} + V_a - V_d = 0 \Rightarrow V_{bh} = V_d - V_a$

Finally, $[V_{bx}] = [A][V_x]$

$$\begin{bmatrix} V_{ba} \\ V_{bb} \\ V_{bc} \\ V_{bd} \\ V_{be} \\ V_{bf} \\ V_{bg} \\ V_{bh} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix}$$

$$[V_{bx}] = [A]^T [V_x]$$

$$[A]^T = \begin{bmatrix} A_b \\ \cdots \\ A_e \end{bmatrix} = \begin{bmatrix} I \\ \cdots \\ A_e \end{bmatrix}$$

V_{bx} → Branch voltage matrix, A → Incidence matrix

V_x → Tree branch matrix.

This matrix is rectangular and therefore singular; Its elements a_{ik} are found as per the following rules.

a_{ik} → 1 if i^{th} element is incident to and oriented away from the k^{th} node.

a_{ik} → −1 if i^{th} element is incident to and oriented towards the k^{th} node.

a_{ik} → 0 if i^{th} element is not incident to the k^{th} node.

Primittive Impedence Matrix [$Z_{primittive}$]

Matrix which contain information about transmission line (impedence) is called primittive impedence matrix.

Size of the primittive impedence matrix is $e \times e$

$e \rightarrow$ Number of elements or branches.

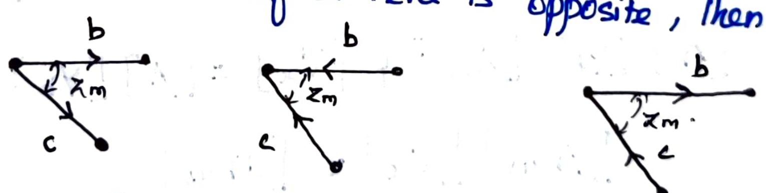
$$Z_{primitive} = \begin{bmatrix} Z_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_b & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_c & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_d & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_f & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_g & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_h \end{bmatrix}$$

If mutual impedence (Z_m) is given, between branches b & c

Assumption: Direction of current is same, then Z_m is +ve

Direction of current is opposite, Then Z_m is −ve

Ex:



$$Z_{primittive} = \begin{bmatrix} Z_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_b & Z_m & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_m & Z_c & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_d & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_f & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_g & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_h \end{bmatrix}$$

Primittive Admittance Matrix

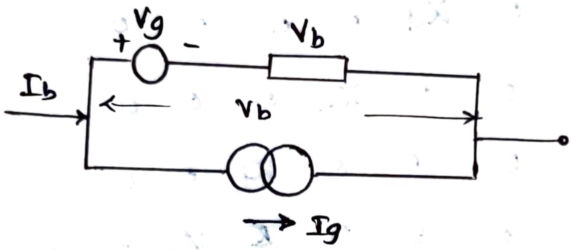
Matrix which contain information about the transmission line (admittance) is called primittive admittance matrix.

$$Y_{primittive} = [Z_{primittive}]^{-1} \quad \text{——— (1)}$$

Bus Admittance Matrix :-

$$[A][I_b] = 0 \quad \text{——— (2)}$$

$$[V_b] = [A]^T [V_x] \quad \text{——— (3)}$$



Using KCL, $I_b = -I_g + Y_b (V_b - V_g)$ ——— (4)

Sub eqn (4) in eqn (2), we get

$$[A][-I_g + Y_b(V_b - V_g)] = 0$$

$$-[A][I_g] + [A][Y_b][V_b] - [A][Y_b][V_g] = 0$$

$$\{[A][Y_b][A]^T\}[V_x] = [A][Y_b][V_g] + [A][I_g]$$

$$[Y_{bus}][V_x] = [I_{bus}]$$

Y_{bus} = Bus Admittance matrix $= [A][Y_b][A]^T$

$= [A][Y_{primittive}][A]^T$

$[V_x]$ = Tree branch voltage matrix.

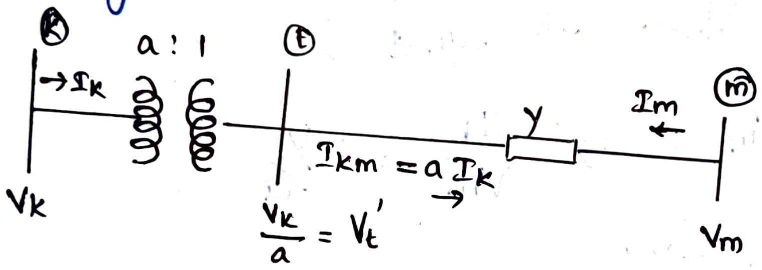
Equivalent Circuit of Transformer with OFF-Nominal-Tap Ratio

The presence of transformer in transmission line modifies bus admittance matrix, thereby modifying the load flow solution.

A two winding transformer with off nominal turns ratio, connected between nodes k and m.

In this representation, the turns ratio is normalized as a : 1 and the non-unity side is called the tap side which is taken as the sending end side.

The series admittance of the transformer is connected to the unity side.



Let us consider the tap ratio as 'a'

$$\frac{V_t'}{V_k} = \frac{1}{a} \Rightarrow V_t' = \frac{V_k}{a} \quad \text{——— (1)}$$

$$\text{Transformation Ratio} = \frac{V_t'}{V_k} = \frac{I_k}{I_{km}} = \frac{1}{a} \quad \text{——— (2)}$$

$$I_{km} = a\, I_k$$

$$\therefore \; I_{km} = (V_t' - V_m)\, y = \left(\frac{V_k}{a} - V_m\right) y \quad \text{——— (3)}$$

$$I_k = \frac{I_{km}}{a} = \left(\frac{V_k}{a^2} - \frac{V_m}{a}\right) y \quad \text{——— (4)}$$

$$I_m = -I_{km} = \left[\frac{-V_k}{a} + V_m\right] y \quad \text{——— (5)}$$

Let us find Y parameters , $Y_{KK} = \left.\frac{I_K}{V_K}\right|_{V_m = 0}$.

Then $V_m = 0$ at eqn ④

$$I_K = \frac{V_K Y}{a^2}. \Rightarrow \frac{I_K}{V_K} = \frac{Y}{a^2}$$

$$Y_{KK} = \frac{Y}{a^2}$$

$$Y_{mm} = \left.\frac{I_m}{V_m}\right|_{V_K = 0}.$$

Substituting $V_K = 0$, at eqn ⑤ $I_m = V_m Y$. $\Rightarrow \frac{I_m}{V_m} = Y$

$$Y_{mm} = Y$$

$$Y_{Km} = \left.\frac{I_K}{V_m}\right|_{V_K = 0}.$$

Substituting $V_K = 0$ at eqn ④ ,

$$I_K = \frac{Y}{a} V_m , \Rightarrow \frac{I_K}{V_m} = -Y/a$$

$$Y_{Km} = -Y/a$$

$$Y_{mK} = \left.\frac{I_m}{V_K}\right|_{V_m = 0}.$$

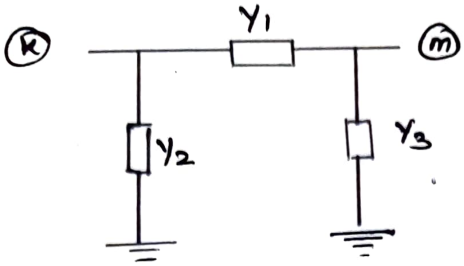
Substituting $V_m = 0$, at eqn ⑤,

$$I_m = \frac{-Y}{a} V_K \Rightarrow \frac{I_m}{V_K} = -Y/a$$

$$Y_{mK} = -Y/a$$

$$\begin{bmatrix} \frac{Y}{a^2} & \frac{-Y}{a} \\ \frac{-Y}{a} & Y \end{bmatrix} \begin{bmatrix} V_K \\ V_m \end{bmatrix} = \begin{bmatrix} I_K \\ I_m \end{bmatrix}$$

convert Y parameters into π - equivalent elements.

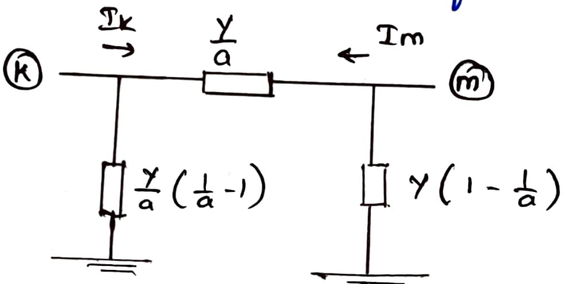


$$Y_1 = -Y_{km} = \frac{y}{a}$$

$$Y_2 = Y_{kk} + Y_{mk} = \frac{y}{a^2} + \left(\frac{-y}{a}\right) = \frac{y}{a}\left[\frac{1}{a} - 1\right]$$

$$Y_3 = Y_{mm} + Y_{km} = y + \left(\frac{-y}{a}\right) = y\left[1 - \frac{1}{a}\right]$$

The π - equivalent circuit for transformer with off nominal tap



UNIT - 2 POWER FLOW ANALYSIS

NEED FOR LOAD FLOW ANALYSIS :-

Load flow analysis is performed on a symmetrical steady state operating condition of a power system under normal mode of operation.

The solution of load flow gives bus voltages and line / transformer power flows for a given load condition.

* Long term plan :-

Load flow analysis helps in investigating the effectiveness of alternative plans and choosing the best plan for system expension to meet the projected operating state.

* Operational planning :-

It helps in choosing the best unit commitment plan. and generation schedules to run the system efficiently for the next day's load condition without violating the bus voltages and line flow operating limit.

steps for load flow study:-

* Representation of the system by single line diagram.
* Determine the impedence diagram using the information in single line diagram.
* Formulation of network equations.
* Solution of network equations.

classification of Buses:-

The power flow equation is

$$P_i + jQ_i = V_i \sum_{j=1}^{N} Y_{ij} * V_{ij}^{*} \quad , i = 1, 2, \ldots N$$

complex bus voltage $V_i = |V_i| \angle \delta_i$

Power system associated with four quantities,

* Real power (P)
* Reactive power (Q)
* Voltage magnitude (|V|)
* phase angle of voltage (δ)

There are three types of buses,

1) slack bus (or) swing bus (or) reference bus
2) Generator bus (or) voltage controlled bus (or) P-V bus (or) regulated bus.
3) Load bus (or) P-Q bus.

Slack Bus :-

In slack bus, voltage magnitude and phase angle of voltages are specified pertaining to a generator bus usually a large capacity generation bus is choosen. We assume Voltage (V) as a reference phasor.

δ - phase angle of voltage = 0.

Power balance equation,

$$\underbrace{P_L}_{\text{Real power loss}} = \sum_{i=1}^{N} P_i = \underbrace{\sum_{i=1}^{N} P_{Gi}}_{\text{Total generation}} - \underbrace{\sum_{i=1}^{N} P_{Di}}_{\text{Total load.}}$$

* P_L depends on I^2R loss in transmission line and transformer of the network.
* The individual currents in the various lines of the network cannot be calculated until after voltage magnitude and angle are known at every bus of the system.
* P_L is initially unknown.
* Real and reactive power are not specified for slack bus.

Generator Bus:-

* The real power and voltages are specified.
* The phase angle of voltages and reactive power are to be determined.
* The limits on the value of reactive power also specified.
* In order to main a good voltage profile over the system Automatic voltage regulation (AVR) is used.
* Static VAR compensator buses are called as P-V buses because real power and voltage magnitude are specified at these buses.

Load Bus:-

At these buses, the active and reactive powers are specified. The magnitude and phase angle of Voltages are unknown. These are called load bus.

DESCRIPITION OF LOAD FLOW PROBLEMS:-

1) Ideal load Flow Problem:-

The network configuration [line impedence and half line charging admittance] and all the bus power injections.

$$P_i = P_G - P_D$$

2) Practical load Flow analysis :-

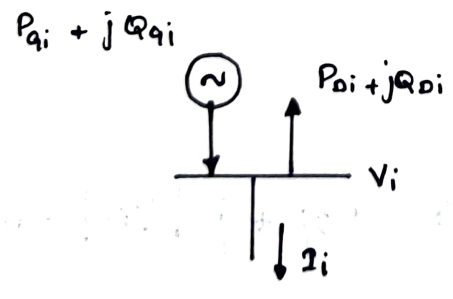
The network configuration, complex power demands for all buses, real power generation schedules and voltage magnitudes of all the P-V buses and voltage magnitude of the slack bus. To determine.

* Bus admittance matrix.
* Bus voltage phase angles of all buses except the slack bus and bus voltage magnitudes of all the P-Q buses.

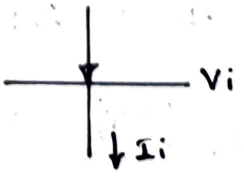
State vector $X = [V_1, V_2 \ldots V_N, \delta_1, \delta_2 \ldots \delta_N]$

Power Flow Equation :- [Development of load flow model in complex variable form and polar variable form].

The power flow or load flow model in complex form is obtained by writing one complex power matching equation at each bus.



$P_i + jQ_i = (P_{gi} - P_{Di}) + j(Q_{gi} - Q_{Di})$



Net power injected into the bus i

$$S_i = S_{gi} - S_{Di}$$

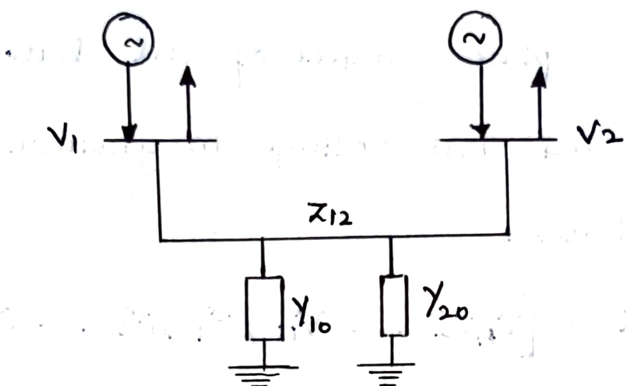
$$S_i = P_{Gi} + j\,Q_{Gi} - (P_{Di} + j\,Q_{Di})$$

$$= P_{Gi} - P_{Di} + j\,(Q_{Gi} - Q_{Di})$$

$$S_i = P_i + j\,Q_i$$

We know that $P_i + j\,Q_i = V_i\, I_i^*$

Two Bus System:-



Let I_1 be the net or bus current entering into bus 1.
Let I_2 be the net current entering into bus 2.

$$[I] = [Y]\,[V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{12} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_{11} = Y_{10} + Y_{12}$$

$$Y_{22} = Y_{20} + Y_{21}$$

$$Y_{12} = Y_{21} = -Y_{21}$$

In general,

$$Y_{ij} = |Y_{ij}| \angle \theta_{ij} = |Y_{ij}| \cos\theta_{ij} + j\,|Y_{ij}| \sin\theta_{ij}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

In general, the net current entering into i^{th} bus

$$I_i = Y_{i1} V_1 + Y_{i2} V_2 \dots Y_{iN} V_N = \sum_{j=1}^{N} Y_{ij} V_j$$

then
$$S_i = P_i + jQ_i = V_i I_i^*$$

$$S_i = P_i - jQ_i = V_i^* I_i$$

$$P_i - jQ_i = V_i^* \sum_{j=1}^{N} Y_{ij} V_j \quad , \text{ where } i = 1, 2 \dots N$$

There are N complex variable equations from which the N unknown complex variables $V_1, V_2, \dots V_N$ can be determined.

$$P_i - jQ_i = V_i^* \sum_{j=1}^{N} |Y_{ij}| \angle \theta_{ij} \,.\, V_j$$

where
$$V_i = |V_i| \angle \delta_i \quad , \quad V_i^* = |V_i| \angle -\delta_i$$

$$V_j = |V_j| \angle \delta_j \quad , \quad \delta_i \rightarrow \text{phase angle of voltage}$$

$$P_i - jQ_i = \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \angle (\theta_{ij} + \delta_j - \delta_i)$$

Equating real & reactive power, we get

$$P_i = \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i = -\sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \sin(\theta_{ij} + \delta_j - \delta_i).$$

Finally,

$$P_i = |V_i|^2 |Y_{ii}| \cos\theta_{ii} + \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i = -|V_i|^2 |Y_{ii}| \sin\theta_{ii} - \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$$

SOLUTION TO LOAD FLOW PROBLEM

The load flow methods are given by,

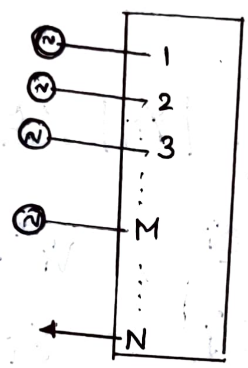
(i) Gauss - Seidel load flow method (GSLF)

(ii) Newton- Raphson load flow method (NRLF)

(iii) Fast Decoupled Load flow method (FDLF)

GAUSS - SEIDEL LOAD FLOW METHOD

This method is also known as successive displacements



Consider 'N' bus system. Bus 1 to M are machine or generator bus. Bus M+1 to N load buses.

Flat voltage start :-

phase angle, $\delta_i^0 = 0$, for $i = 1, 2, \ldots N$ (except slack)

Voltage $|V_i^0| = 1.0$, for $i = M+1 \ldots N$ (for P-V buses)

Bus 1 is a generator bus and take it as reference bus or slack bus. Here the voltage is specified.

In load buses, assume initial value of voltage as $1\angle 0^\circ$ and find the new value of voltages.

In bus 2, the generator buses, first check for generator limit and find the voltages, Injected bus power is given by,

$$S_i = P_i - jQ_i = V_i^* I_i$$
$$= V_i^* \sum_{j=1}^{N} Y_{ij} V_j$$

$$P_i - jQ_i = V_i^* Y_{ii} V_i + V_i^* \sum_{\substack{j=1 \\ \neq i}}^{N} Y_{ij} V_j$$

$$V_i^* Y_{ii} V_i = P_i - jQ_i - V_i^* \sum_{\substack{j=1 \\ \neq i}}^{N} Y_{ij} V_j$$

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ \neq i}}^{N} Y_{ij} V_j \right]$$

$i = 1, 2, 3 \dots N$ except slack bus.

Let $V_1^{old}, V_2^{old} \dots V_N^{old}$ be the initial bus voltages.

On substituting initial values in the above equation, we can find $V_2^{new}, V_3^{new} \dots V_N^{new}$. After calculating each voltages replace the old bus value by new values.

$$V_i^{new} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^{*old}} - \sum_{j=1}^{i-1} Y_{ij} V_j^{new} - \sum_{j=i+1}^{N} Y_{ij} V_j^{old} \right]$$

* For load bus, the above equation is applicable to find $|V|$ and δ values.

* For slack bus, so it will not change in each iteration

For PV or generator bus,

(i) Q value is not specified for PV bus, Adjusting the complex $V_i = e_i + j f_i$ to correct the voltage magnitude to the specified value $|V_i|_{spec}$.

$$V_i^{new} = |V_i|_{spec} \angle \delta^{cal}$$

$$\delta_{cal} = \tan^{-1}\left[\frac{f_i}{e_i}\right]$$

(ii) Compute the reactive power generation using V_i^{new}

$$Q_i^{cal} = -\,\mathrm{Im}\left\{ V_i^{old*}\left[\sum_{j=1}^{i-1} Y_{ij} V_j^{new} + \sum_{j=1}^{N} Y_{ij} V_j^{old}\right]\right\}$$

$$Q_{Gi} = Q_i^{cal} + Q_{Di}$$

If $Q_{Gi(min)} \le Q_{Gi} \le Q_{Gi(max)}$, set $Q_i = Q_{Gi} - Q_{Di}$ then compute V_i^{new},

If $Q_{Gi} < Q_{Gi(min)}$, Set $Q_{Gi} = Q_{Gi(min)}$, then compute V_i^{new}

If $Q_{Gi} > Q_{Gi(max)}$, set $Q_{Gi} = Q_{Gi(max)}$, then compute V_i^{new}

Acceleration Factor (α) :-

In Gauss Seidal Method, the number of iterations required for convergence can be reduced, if the correction in bus voltage computed at each iteration is multiplied by a factor greater than unity, called as acceleration factor to bring the voltage closer than to the value to which it is converging.

The range of 1.3 to 1.7 is found to be satisfactory for typical systems,

$$V_i^{new} = V_i^{old} + \alpha \left[V_i^{new} - V_i^{old} \right]$$

$V_i^{old} \rightarrow$ Voltage value obtained in the previous iteration

$\alpha \rightarrow$ acceleration factor

$V_i^{new} \rightarrow$ New value of voltage obtained in the current iteration

Convergence Check :-

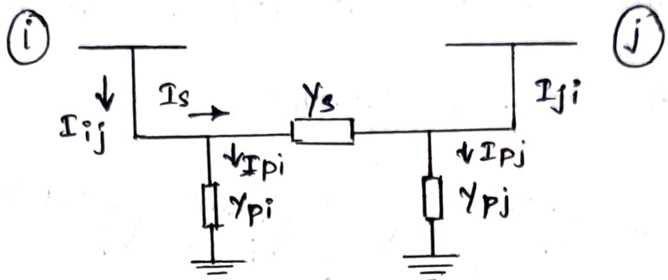
* For the power mismatch is small and acceptable, a very tight tolerance must be specified on both real and imaginary components of voltage.
* Iteration process continuous untill the magnitude ΔP and $\Delta Q < 0.001$ p.u (specified value).
* Voltage accuracy is 0.00001

$$\Delta V_{max} = \max \{ \Delta e_k + j \Delta f_k \}, \quad k = 1, 2 \cdots N \neq slack.$$

(i) Computation of Slack Bus Power :-

Slack bus power $P_1 - jQ_1 = V_1^* \sum_{j=1}^{N} Y_{ij} V_j$

(ii) Computation of Line Flows :-



Consider the line connecting two buses i and j. The line can be represented by the series admittance Y_s and the two shunt admittances (half line charging admittance) Y_{pi} and Y_{pj}.

Line current (forward) $I_{ij} = I_s + I_{pi}$

$$I_{ij} = (V_i - V_j) Y_s + V_i \times Y_{pi}$$

Line current (reverse) $I_{ji} = -I_s + I_{pj}$

$$I_{ji} = (V_j - V_i) Y_s + V_j \times Y_{pj}$$

Line power (forward) $S_{ij} = P_{ij} + jQ_{ij} = V_i I_{ij}^*$

$$S_{ij} = V_i \left[(V_i - V_j) Y_s + V_i Y_{pi} \right]^*$$

$$= V_i \left[(V_i^* - V_j^*) Y_s^* + |V_i|^2 Y_{pi}^* \right]$$

Line power (reverse) $S_{ji} = V_j I_{ji}^*$

$$S_{ji} = V_j \left[(V_j^* - V_i^*) Y_s^* + |V_j|^2 Y_{pj}^* \right]$$

(iii) Computation of Transmission Loss :-

power loss in the transmission line ij

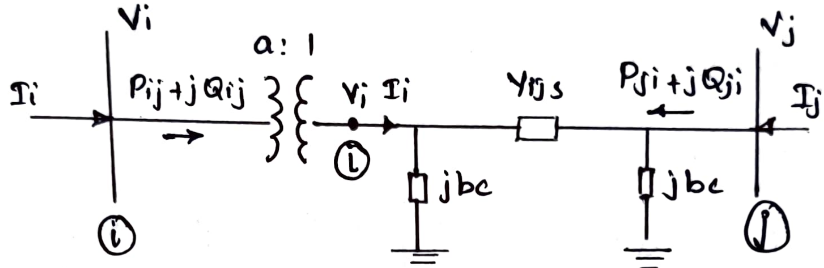
$$S_{ij}(\text{loss}) = S_{ij} + S_{ji}$$

$$= P_{ij} + jQ_{ij} + P_{ji} + jQ_{ji}$$

Real power = $P_{ij} + P_{ji}$

Reactive power = $Q_{ij} + Q_{ji}$

(iv) Computation of transformer and line flow equation :-



After finding complex bus voltages, the active and reactive flows in all the line / transformers are to be computed. A common π equivalent circuit for transmission line and transformer.

For line $a = 0$,

For transformer $b_c = 0$

Power flow from i^{th} bus to the j^{th} bus, measured at the i^{th} bus is given by

$$P_{ij} + jQ_{ij} = V_i I_i^* = V_t^* I_t^*$$

$$\frac{V_i}{V_t} = a \Rightarrow V_t = \frac{V_i}{a}$$

$$I_t = (V_t - V_j)\, Y_{ijs} + V_i (j b_c)$$

then,
$$P_{ij} + jQ_{ij} = \frac{V_i}{a}\left[\left[\frac{V_i^*}{a} - V_j^*\right] Y_{ijs}^*\right] + \left|\frac{V_j}{a}\right|^2 (j b_c)^*$$

Similarly power from j^{th} bus to i^{th} bus,

$$P_{ji} + jQ_{ji} = V_j I_j^* = V_j\left[(V_j^* - V_t^*)\, Y_{ijs}\right] + V_j^2 (j b_c)^*$$

$$= V_j\left[V_j^* - \left(\frac{V_i^*}{a}\right)\right] Y_{jis}^* + V_j^2 (j b_c)^*$$

For Transformer :-

Now $b_c = 0$,

$$P_{ij} + jQ_{ij} = \frac{V_i}{a}\left[\left(\frac{V_i^*}{a} - V_j^*\right) Y_{ijs}^*\right]$$

$$P_{ji} + jQ_{ji} = V_j\left[V_j^* - \left(\frac{V_i^*}{a}\right)\right] Y_{jis}^*$$

Real power loss $P_{loss} = P_{ij} + P_{ji}$

Reactive power loss $Q_{Loss} = Q_{ij} + Q_{ji}$

Algorithm For Iteration Method :-

① Form Y-bus matrix

② Assume $V_k = V_{k(spec)} \angle 0^\circ$ at all generator buses.

③ Assume $V_k = 1\angle 0^\circ = 1 + j0$ at all load buses.

④ Set iteration count = 1

⑤ Let bus number $i = 1$

⑥ If 'i' refers to generator bus go to step 7, otherwise go to step 8.

⑦ a) If 'i' refers to the slack bus go to step 9, otherwise go to step 7(b)

b) compute Q_i using

$$Q_i^{cal} = -\operatorname{Im}\left[\sum_{j=1}^{N} V_i^{*} Y_{ij} V_j\right]$$

$$Q_{Gi} = Q_i^{cal} + Q_{Li}$$

check for Q limit violation,

If $Q_{i(min)} < Q_{Gi} < Q_{i(max)}$, then $Q_{i(spec)} = Q_i^{cal}$

If $Q_{i(min)} < Q_{Gi}$, then $Q_{i(spec)} = Q_{i(min)} - Q_{Li}$

If $Q_{i(max)} < Q_{Gi}$, then $Q_{i(spec)} = Q_{i(max)} - Q_{Li}$

If Q_{limit} is violated, then treat this bus as P-Q bus till convergence is obtained.

⑧ Compute V_i using the equation,

$$V_i^{new} = \frac{1}{Y_{ii}}\left[\frac{P_{i(spec)} - Q_{i(spec)}}{V_i^{old*}} - \sum_{i=1}^{j-1} Y_{ij} V_j^{new} - \sum_{i=j+1}^{N} Y_{ij} V_i^{old}\right]$$

⑨ If i is less than number of buses, increment 'i' by 1 and go to step 6.

(10) compare two successive iteration values for V_i

If $V_i^{new} - V_i^{old}$ < tolarance, go to step 12.

(11) Update the new voltage as,

$$V^{new} = V^{old} + \alpha \left(V^{new} - V^{old} \right)$$

$$V^{old} = V^{new}$$

iter = iter + 1 ; go to step 5

(12) compute relevant quantities,

Slack bus power $S_1 = P_i - jQ_i = V * I$

$$S_1 = V_i^* \sum_{j=1}^{N} Y_{ij} V_j$$

Line flows $S_{ij} = P_{ij} + j\, Q_{ij}$

$$S_{ij} = V_i \left[V_i^* - V_j^* \right] Y_{ij\,series}^* + |V_i|^2 Y_{ii}^*$$

$$P_{Loss} = P_{ij} + P_{ji}$$

$$Q_{Loss} = Q_{ij} + Q_{ji}$$

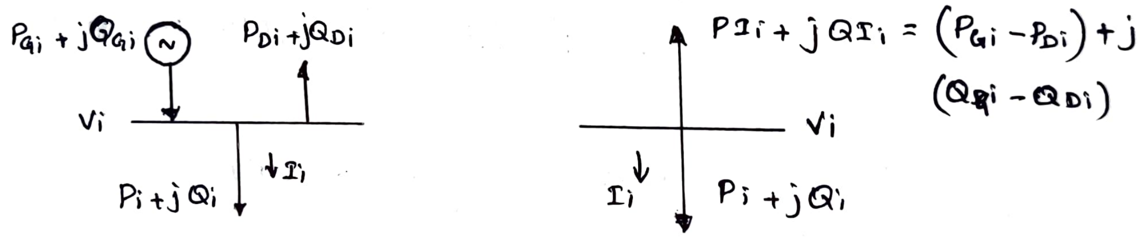
(13) Stop the execution.

NEWTON - RAPHSON METHOD

Advantage over Gauss Seidal Method :-

* converges equally fast for large as well as small systems.
* Less than 4 to 5 iterations is needed.
* more functional evaluations are required.
* Very popular for large system.
* Non linear algebraic equations are solved.
* Based on successive approximation procedure on an initial estimate of the unknown and Taylor's series expansion.

Load Flow Model in Real Variable Form :-



The complex power balance at bus i is given by,

$$PI_i + jQI_i = P_i + jQ_i \quad \text{——— (1)}$$

complex power injection at the i^{th} bus

$$PI_i + jQI_i = (P_{Gi} - P_{Di}) + j(Q_{Gi} - Q_{Di})$$

Since the generation bus and demand are specified, the complex power injection is a specified quantity and is given by

$$PI_{i(spec)} + j\,QI_{i(spec)} = \left[P_{Gi(spec)} - P_{Di(spec)}\right] + j\left[Q_{Gi(spec)} - Q_{Di(spec)}\right]$$

The current entering bus i is given by ——— ②

$$I_i = \sum_{j=1}^{N} Y_{ij} V_j$$

In polar form, $I_i = \sum_{j=1} |Y_{ij}|\,|V_j| \angle (\theta_{ij} + \delta_j)$

$$Y_{ij} = |Y_{ij}| \angle \theta_{ij} \quad ; \quad V_i = |V_i| \angle \delta_i \qquad \text{—— ③}$$

complex power at bus i,

$$P_i - jQ_i = V_i^* I_i = V_i^* \sum_{j=1}^{N} Y_{ij} V_j$$

substituting from eqn ③,

$$P_i - jQ_i = |V_i| \angle{-\delta_i} \sum_{j=1}^{N} |Y_{ij}|\,|V_j| \angle (\theta_{ij} + \delta_j)$$

$$= \sum_{j=1}^{N} |V_i|\,|Y_{ij}|\,|V_j| \angle (\theta_{ij} + \delta_j - \delta_i)$$

Equating real and imaginary values,

$$P_i = \sum_{j=1}^{N} |V_i|\,|Y_{ij}|\,|V_j| \cos(\theta_{ij} + \delta_j - \delta_i) \qquad \text{—— ④}$$

$$Q_i = \sum_{j=1}^{N} |V_i|\,|Y_{ij}|\,|V_j| \sin(\theta_{ij} + \delta_j - \delta_i). \qquad \text{—— ⑤}$$

Power Balance Equation

$P_i(\delta, V) - PI_{i(spec)} = 0$, $i = 1, 2 \ldots N \neq$ slack

δ – unknown.

$Q_i(\delta, V) - QI_{i(spec)} = 0$, $i = 1, 2 \ldots (N - M - 1)$

$|V| \rightarrow$ unknown.

$M \rightarrow$ no. of P-V buses.

Newton Raphson Load Flow Algorithm Including PV bus Adjustment :-

Let us assume all the buses are load bus except Slack bus. The unknown variables consists of $|V_2|, |V_3| \ldots$ $\ldots |V_n|$ and voltage angles are $\delta_2, \delta_3 \ldots \delta_N$.

Initial guess of state Vector $[x^0] = \begin{bmatrix} \delta_2^0 \\ \delta_3^0 \\ \vdots \\ \delta_N^0 \\ V_2^0 \\ V_3^0 \\ \vdots \\ V_N^0 \end{bmatrix}$

Using Taylor series, neglecting higher order terms,

$$F(x^0 + \Delta x) = 0 \approx F(x^0) + \left.\frac{\delta F}{\partial x}\right|_{x^0} \cdot \Delta x$$

$$P_i \approx P_i^0 + \left[\frac{\partial P_i}{\partial \delta_2}\right]^0 \cdot \Delta \delta_2^0 + \cdots + \left[\frac{\partial P_i}{\partial \delta_N}\right] \cdot \Delta \delta_N^0 + \left[\frac{\partial P_i}{\partial |V_2|}\right]^0 |\Delta V_2^0| + \cdots$$
$$+ \left[\frac{\partial P_i}{\partial |V_N|}\right]^0 \cdot |\Delta V_N^0|$$

for $i = 1, 2, 3 \ldots N$

$$Q_i \approx Q_i^0 + \left[\frac{\partial Q_i}{\partial \delta_2}\right]^0 \cdot \Delta \delta_2^0 + \cdots + \left[\frac{\partial Q_i}{\partial \delta_N}\right]^0 \cdot \Delta \delta_N + \left[\frac{\partial Q_i}{\partial |V_2|}\right]^0 \cdot |\Delta V_2^0|$$
$$+ \cdots + \left[\frac{\partial Q_i}{\partial |V_N|}\right]^0 \cdot |\Delta V_N^0|$$

for $i = M+1, \ldots N$

Real power mismatch $\Delta P_i^{\circ} \approx P_i - P_i^{\circ}$

Reactive power mismatch $\Delta Q_i^{\circ} \approx Q_i - Q_i^{\circ}$

In matrix form,

$$\begin{bmatrix} \Delta P_2^{\circ} \\ \vdots \\ \Delta P_N^{\circ} \\ \Delta Q_2^{\circ} \\ \vdots \\ \Delta Q_N^{\circ} \end{bmatrix} = \left[\begin{array}{ccc|ccc} \left(\frac{\partial P_2}{\partial \delta_2}\right)^{\circ} & \cdots & \left(\frac{\partial P_2}{\partial \delta_N}\right)^{\circ} & \left(\frac{\partial P_2}{\partial |V_2|}\right)^{\circ} & \cdots & \left(\frac{\partial P_2}{\partial |V_N|}\right)^{\circ} \\ \vdots & & \vdots & \vdots & & \vdots \\ \left(\frac{\partial P_N}{\partial \delta_2}\right)^{\circ} & \cdots & \left(\frac{\partial P_N}{\partial \delta_N}\right)^{\circ} & \left(\frac{\partial P_N}{\partial |V_2|}\right)^{\circ} & \cdots & \left(\frac{\partial P_N}{\partial |V_N|}\right)^{\circ} \\ \hline \left(\frac{\partial Q_2}{\partial \delta_2}\right)^{\circ} & \cdots & \left(\frac{\partial Q_2}{\partial \delta_N}\right)^{\circ} & \left(\frac{\partial Q_2}{\partial |V_2|}\right)^{\circ} & \cdots & \left(\frac{\partial Q_2}{\partial |V_N|}\right) \\ \vdots & & \vdots & \vdots & & \vdots \\ \left(\frac{\partial Q_N}{\partial \delta_2}\right)^{\circ} & \cdots & \left(\frac{\partial Q_N}{\partial \delta_N}\right)^{\circ} & \left[\frac{\partial Q_N}{\partial |V_2|}\right]^{\circ} & \cdots & \left(\frac{\partial Q_N}{\partial |V_N|}\right) \end{array}\right]$$

Simply $\quad [\Delta u^{\circ}] = [J^{\circ}] \cdot [\Delta x^{\circ}]$

$[J^{\circ}] \rightarrow$ Jacobian matrix.

Bus 1 $\rightarrow$ slack bus.

The Jacobian matrix gives the linearized relationship between small changes in voltage angle $\Delta \delta_i^{\circ}$ and voltage magnitude $|\Delta V_i^{\circ}|$ with small change in real and reactive power ΔP_i° and ΔQ_i°

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

The diagonal and off diagonal elements of J_1 are

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ \neq i}}^{N} |V_i| |Y_{ij}| |V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial P_i}{\partial \delta_j} = - |V_i| |Y_{ij}| |V_j| \sin(\theta_{ij} + \delta_j - \delta_i).$$

The diagonal and off diagonal elements of J_2 are

$$\frac{\partial P_i}{\partial |V_i|} = 2 |V_i| |Y_{ii}| \cos\theta_{ii} + \sum_{\substack{j=1 \\ i \neq j}}^{N} |V_j| |Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i| |Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i)$$

The diagonal and off diagonal elements of J_3 are

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ \neq i}}^{N} |V_i| |Y_{ij}| |V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial Q_i}{\partial \delta_j} = - |V_i| |Y_{ij}| |V_j| \cos(\theta_{ij} + \delta_j - \delta_i).$$

The diagonal and off diagonal elements of J_4 are,

$$\frac{\partial Q_i}{\partial |V_i|} = -2 |V_i| |Y_{ii}| \sin\theta_{ii} - \sum_{\substack{j=1 \\ \neq i}}^{N} |V_j| |Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial Q_i}{\partial |V_j|} = - |V_i| |Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i)$$

The terms ΔP_i and ΔQ_i are the difference between the specified and calculated values known as the power residues, given by

$$\Delta P_i = P_{i\,(spec)} - P_i^{cal}$$

$$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$$

The new estimates for bus voltages are

$$\delta_i^{new} = \delta_i^{old} + \Delta\delta_i^{old}$$

$$V_i^{new} = V_i^{old} + \Delta|V_i|^{old}$$

For PV buses or voltage controlled buses :-

* The voltage magnitudes are specified for PV bus.
* Let M be the number of generator buses. M equations involving ΔQ and ΔV and the corresponding columns of the Jacobian matrix are eliminated.
* There are (N-1) real power constrains and (N-1-M) reactive power constraints and Jacobian matrix of the order $(2N-2-M) * (2N-2-M)$.

Algorithm :-

1) Formulate Y-bus matrix
2) Assume flat start for starting voltage solution,

$\delta_i^0 = 0$, for $i = 1, 2 \ldots N$ for all buses except slack

$|V_i|^0 = 1.0$, for $i = M+1, M+2 \ldots N$ (all PQ buses)

$|V_i| = |V_i|_{(spec)}$, for all PV buses and slack bus.

3) For load buses, calculate P_i^{cal} and Q_i^{cal}.

4) For PV buses, check for Q - limit Violation

if $Q_{i(min)} < Q_i^{cal} < Q_{i(max)}$, the bus acts as PV bus.

if $Q_i^{cal} > Q_{i(max)}$, $Q_{i(spec)} = Q_{i(max)}$

if $Q_i^{cal} < Q_{i(min)}$, $Q_{i(spec)} = Q_{i(min)}$

Now PV bus act as PQ bus.

5) Compute mismatch Vector using

$$\Delta P_i = P_{i(spec)} - P_i^{cal}$$

$$\Delta Q_i = Q_{i(spec)} - Q_i^{cal}$$

6) compute

$$\Delta P_{i(max)} = \max |\Delta P_i| \; ; \text{ for } i = 1, 2 \cdots N \text{ except slack}$$

$$\Delta Q_{i(max)} = \max |\Delta Q_i| \; , \text{ for } i = M+1 \cdots N .$$

7) Compute Jacobian matrix using

$$J = \begin{bmatrix} \frac{\partial P_i}{\partial \delta} & \frac{\partial P_i}{\partial |V|} \\ \frac{\partial Q_i}{\partial \delta} & \frac{\partial Q_i}{\partial |V|} \end{bmatrix}$$

8) Obtain state correction Vector

$$\begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

10) This procedure is continued until

$$|\Delta P_i| < \varepsilon \quad \text{and} \quad |\Delta Q_i| < \varepsilon$$

otherwise goto step 3.

Comparison of Gauss Seidal and Newton Raphson Method:

1) For G-S method, the variables are expressed in rectangular form coordinates whereas N-R method, they are expressed in polar coordinates because memory requirement will be more.

2) For G-S method, mathematical operation per iteration is ~~less~~ more compare to N-R method.

3) G-S method → linear convergence characteristics
N-R method → quadratic convergence characteristic.
So N-R method is faster.

4) G-S method → No of iteration increases due to increasing buses.
N-R method → Iteration remains constant. It doesnot depends on buses.

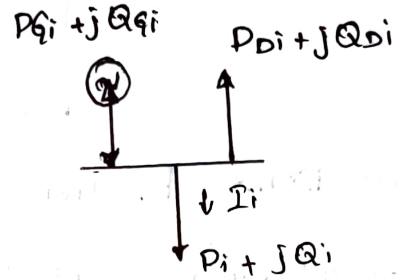
5) G-S method → convergence is affected by slack bus choosen and presence of series capacitors
N-R method → less sensitive for these factors.

6) G-S method → need more iteration (more than 30)
N-R method → less than 5 iteration needed.

Fast Decoupled Power Flow (FDPF).

Advantages :-

* It is faster , simple to program , more reliable and requires less memory than NR load flow method.
* It requires more iteration compare to NR method. But requires less time per each iteration.



The complex power injection at the i^{th} bus,

$$P_i + jQ_i = (P_{Gi} - P_{Di}) + j(Q_{Gi} - Q_{Di})$$

Current entering the bus 'i',

$$I_i = \sum_{j=1}^{N} Y_{ij} \cdot V_j$$

$$= \sum_{j=1}^{N} |Y_{ij}||V_j| \angle \theta_{ij} + \delta_j$$

$$P_i - jQ_i = V_i^* I_i$$

$$= |V_i| \angle -\delta_i \sum_{j=1}^{N} |Y_{ij}||V_j| \angle \theta_{ij} + \delta_j$$

$$= \sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \angle \theta_{ij} + \delta_j - \delta_i$$

$$P_i = \sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i = -\sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$$

where $|V|$ in per unit and phase angle in radians.

Assume all are load bus except slack bus.

$$\text{Initial Guess of State vector } [x^0] = \begin{bmatrix} \delta_2^0 \\ \delta_3^0 \\ \vdots \\ \delta_N^0 \\ V_2^0 \\ V_3^0 \\ \vdots \\ V_N^0 \end{bmatrix}$$

Expanding equations P_i and Q_i using Taylor's series,

[δ_i for all buses except for slack bus ; V_i for load bus]

$$P_i = P_i^0 + \left[\frac{\partial P_i}{\partial \delta_2}\right] \cdot \Delta\delta_2^0 + \dots + \left[\frac{\partial P_i}{\partial \delta_N}\right] \Delta\delta_N^0 + \left[\frac{\partial P_i}{\partial |V_2|}\right] |\Delta V_2^0| + \dots + \frac{\partial P_i}{\partial |V_N|} \cdot |\Delta V_N^0| \quad (1)$$

$$Q_i = Q_i^0 + \left[\frac{\partial Q_i}{\partial \delta_2}\right] \cdot \Delta\delta_2^0 + \dots + \left[\frac{\partial Q_i}{\partial \delta_N}\right] \cdot \Delta\delta_N^0 + \frac{\partial Q_i}{\partial |V_2|} \cdot |\Delta V_2^0| + \dots + \frac{\partial Q_i}{\partial |V_N|} |\Delta V_N^0| \quad (2)$$

In matrix method

$$\begin{bmatrix} \Delta P_2^0 \\ \vdots \\ \Delta P_N \\ \Delta Q_2^0 \\ \vdots \\ \Delta Q_N \end{bmatrix} = \left[\begin{array}{cccc|cccc} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2^0}{\partial \delta_3} & \dots & \frac{\partial P_2^0}{\partial \delta_N} & \frac{\partial P_2^0}{\partial |V_2|} & \frac{\partial P_2^0}{\partial |V_3|} & \dots & \frac{\partial P_2^0}{\partial |V_N|} \\ \vdots & & & \vdots & & & & \\ \frac{\partial P_N}{\partial \delta_2} & \frac{\partial P_N}{\partial \delta_3} & \dots & \frac{\partial P_N^0}{\partial \delta_N} & \frac{\partial P_N}{\partial |V_2|} & \frac{\partial P_N}{\partial |V_3|} & \dots & \frac{\partial P_N^0}{\partial |V_N|} \\ \hline \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \dots & \frac{\partial Q_2}{\partial \delta_N} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} & \dots & \frac{\partial Q_2^0}{\partial |V_N|} \\ \vdots & & & \vdots & \vdots & & & \\ \frac{\partial Q_N}{\partial \delta_2} & \frac{\partial Q_N}{\partial \delta_3} & \dots & \frac{\partial Q_N}{\partial \delta_N} & \frac{\partial Q_N}{\partial |V_2|} & \frac{\partial Q_N}{\partial |V_3|} & \dots & \frac{\partial Q_N}{\partial |V_N|} \end{array}\right] \begin{bmatrix} \Delta\delta_2 \\ \vdots \\ \Delta\delta_N \\ |\Delta V_2| \\ \vdots \\ |\Delta V_N| \end{bmatrix} \quad (3)$$

The compact form,

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \quad \text{——— (3)}$$

In transmission line have a very high $\frac{X}{R}$ ratio,

$\therefore$ Real power change $\Delta P \propto \Delta \delta$

Reactive power change $\Delta Q \propto \Delta V$

Applying $P-\delta$ and $Q-V$ decoupling principle,

$\therefore$ Set $J_2, J_3 = 0$

$$\therefore \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

$$\Delta P = J_1 \Delta \delta = \left(\frac{\partial P}{\partial \delta} \right) \Delta \delta$$

$$\Delta Q = J_4 \Delta |V| = \frac{\partial Q}{\partial |V|} \cdot \Delta |V|$$

Diagonal element $J_1 = \frac{\partial P_i}{\partial \delta_i}$

Differentiate eqn (1), w.r.to δ_i

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ \neq i}}^{N} |V_i| |Y_{ij}| |V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$$

$$= \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \sin(\theta_{ij} + \delta_j - \delta_i) - |V_i|^2 |Y_{ii}| \sin \theta_{ii}$$

Sub eqn (2) for Q_i

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2 |Y_{ii}| \sin \theta_{ii}$$

$$= -Q_i - |V_i|^2 B_{ii}$$

Assume, Imaginary part of $Y_{ii} = B_{ii} = Y_{ii} \sin\theta_{ii}$

$B_{ii} >> Q_i$, we can neglect Q_i

$|V_i|^2 \approx |V_i|$

$$\therefore \frac{\partial P_i}{\partial \delta_i} = -|V_i| B_{ii} \Rightarrow \frac{\Delta P_i}{|V_i|} = -B_{ii} \Delta \delta_i \quad \text{—— (4)}$$

Off diagonal element of $J_1 = \frac{\partial P_i}{\partial \delta_j}$

$$= -|V_i| |Y_{ij}| |V_j| \sin(\theta_{ij} - \delta_j - \delta_i)$$

Value of $\delta_j - \delta_i$ is very small, $|V_j| \approx 1$

$$\therefore \frac{\partial P_i}{\partial \delta_j} = -|V_i| |Y_{ij}| |V_j| \sin\theta_{ij}$$

$$= -|V_i| B_{ij} \quad \text{—— (5)}$$

Diagonal element of $J_4 = \frac{\delta Q_i}{\delta |V_i|}$

Differentiate Q_1 from equation 2 w.r.to $|V_i|$,

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}| \sin\theta_{ii} - \sum_{\substack{j=1 \\ \neq i}}^{N} |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_j - \delta_i)$$

$$= -|V_i||Y_{ii}| \sin\theta_{ii} - \sum_{j=1}^{N} |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_j - \delta_i) \quad \text{—— (6)}$$

Multiplying eqn (6) w.r. to $|V_i|$

$$|V_i| \frac{\partial Q_i}{\partial |V_i|} = -|V_i|^2 |Y_{ii}| \sin \theta_{ii} - \sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \sin(\theta_{ij}+\delta_j-\delta_i)$$

$$= -|V_i|^2 B_{ii} - Q_i$$

$$|V_i| \frac{\partial Q_i}{\partial |V_i|} = -B_{ii} |V_i|^2 \qquad \because B_{ii} >> Q_i$$

$$\frac{\partial Q_i}{\partial |V_i|} = -B_{ii} |V_i| \quad \text{———— (7)}$$

Off diagonal element of $J_4 = \frac{\partial Q_i}{\partial |V_j|}$

Differentiate Q_i from eqn (2), w.r.t V_j

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i| |Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i| B_{ij} \quad \text{———— (8)}$$

Now,

$$\left.\begin{aligned} \frac{\Delta P}{\Delta \delta} &= -|V_i| B_{ij} = -B' |V_i| \\ \Rightarrow \frac{\Delta P}{|V_i|} &= -B' \Delta\delta \\ \frac{\Delta Q}{\Delta |V_i|} = B'' |V_i| \Rightarrow \frac{\Delta Q}{\Delta |V_i|} &= -B'' \Delta |V_i| \end{aligned}\right\} \text{(9)}$$

$B' \rightarrow$ imaginary part of Y bus matrix for all the buses except slack bus.

$B'' \rightarrow$ imaginary part of Y bus matrix for all load buses.

Algorithm :-

1) Formulate Y bus matrix, then compute bus susceptance matrix B' and B''

2) Assume flat start of starting voltage solution

$\delta_i^0 = 0$, for $i = 1, 2 \ldots N$, for all buses excep slack

$|V_i|^0 = 1.0$ for $i = M+1 \ldots N$, for all P-Q buses.

$|V_i| = |V_i|_{(spec)}$ for all P-V Buses and slack bus

3) For load buses, calculate P_i^{cal} & Q_i^{cal} using,

$$P_i^{cal} = \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i^{cal} = \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$$

4) For P-V buses, check for Q limit,

If $Q_{i(min)} \leq Q_i \leq Q_{i(min)}$, calculate P_i^{cal}

If $Q_i^{cal} < Q_{i(min)}$, $Q_{i(spec)} = Q_{i(min)}$

If $Q_i^{cal} > Q_{i(max)}$, $Q_{i(spec)} = Q_{i(max)}$

Now PV bus act as a P-Q bus.

Calculate P_i^{cal} & Q_i^{cal}.

UNIT - 3 SYMMETRICAL FAULT ANALYSIS

* Fault analysis is an important part of power system analysis.
* Short circuit studies are performed to determine bus voltages and current flowing in the lines during various types of fault.

  * Symmetrical or balanced faults
  * Unsymmetrical or unbalanced faults.
    * L - G (Line to ground faults)
    * L - L (Line to line faults)
    * L - L - G (Double line to ground fault).

When network is symmetrical fault, the phase currents and phase voltages possess three phase symmetry. ie equal in magnitude and phase shift.

For symmetrical,

$$\text{Fault current } (I_f) = \frac{E_{Th}}{Z_{Th} + Z_f}$$

Z_{Th} → Thevenin's impedence

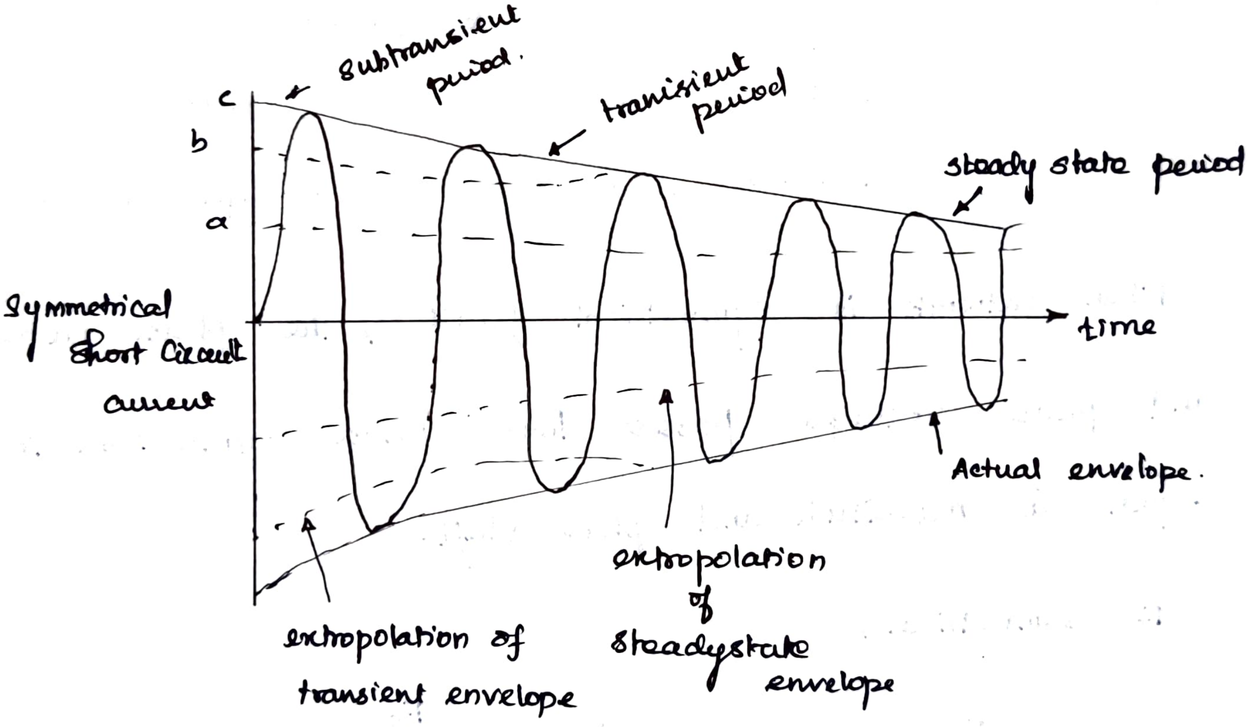
Z_f → Fault impedence

E_{Th} → Thevenin's voltage (or) pre-fault voltage.

* The magnitude of fault current depends on the internal impedence (Z_{Th}) of generator and the fault impedence (Z_f)

* The internal impedence of generator, ~~but~~ under short circuit condition is not constant.

* For a 3ϕ short circuit occurs on the unloaded generator, behaviour can be divided into three periods.



Subtransient Period :- This period is lasting only for the first few cycles.

Transient Period :- After subtransient period, transient period covers a long time.

Steady state period :- After transient period, the system is steady state finally.

Short circuit studies are used -

(i) Proper relay setting and co ordination

(ii) to obtain the rating of the protective switchgears

(iii) to select the circuit breakers.

(iv) to perform whenever system expansion is planned.

(v) to select and set phase relays, while the line to ground fault is used for earth relays, a 3φ fault information is used.

SOLID Fault or Bolted Fault :-

A fault represents a structural network change equivalent with that caused by the addition of an impedence at the place of fault. If the fault impedence is zero, Then the fault is called as bolted or solid fault.

Need For Short Circuit Study :-

* The system must be protected against heavy flow of short circuit currents by dis connecting the faulty section from the healthy section by means of circuit breaker.

* To estimate the magnitude of fault current for the proper choice of circuit breaker and protective relays, short circuit is essential.
* To design a protective schemes, short circuit study is important.

Causes of Symmetrical Fault :-

* Due to insulation failure of equipment
* flash over of lines initiated by a lightning stroke.
* Accidental in power system.
* Due to slow fault clearance, an earth fault spreads across to the other two phases
* Due to induced rotor current decays more rapidly than others.
* Due to short circuit in synchronous generator, the induced rotor current increases. The circuit model of the machine change for subtransient, transient and steady state per[illegible]
* The machine parameters that influence rapidly decaying component are called the transient parameters and those influencing sustained components are the synchronous parameters. Similarly the reactance parameters also defined.
* Selection of circuit breaker depends on initial and transi[illegible] time of current flow to the circuit.

* Both generator and motor subtransient reactances are used to determine the momentry current flowing on occurance of a short circuit.

* Subtransient reactances used for generator and transient reactance is used for synchronous motor.

Basic Assumption in Fault Analysis

1) Representing each machine by a constant voltage source behind proper rectances which may be x'' (Subtransient reactance, x' (transient reactance) or x (Steady State reactance).

2) Pre fault load currents are neglected

3) Transformer taps are assumed to be neglected.

4) Shunt elements in transformer model that account for magnetizing current and core loss are neglected.

5) A symmetric three phase power system is considered.

6) Shunt Capacitance of transmission line are ignored.

7) Series resistances of transmission lines are neglected.

8) The negative sequence impedence of alternators are assumed to be same as that of positive sequence impedence. $Z^+ = Z^-$

Types of Fault :-

A fault in a circuit is any failure which interfaces with the normal flow of current. The fault can be classified.

* Shunt fault (Short circuit fault)
* Series fault (open circuit fault).

Shunt Fault :-

* Three phase fault (LLLG fault) :- symmetrical fault
* Line to ground fault (LG fault)
* Line to line fault (LL fault)
* Double line to ground fault (LLG fault)

unsymmetrical fault

Shunt faults are characterised by increasing current and fall in voltage and frequency.

Series Fault :-

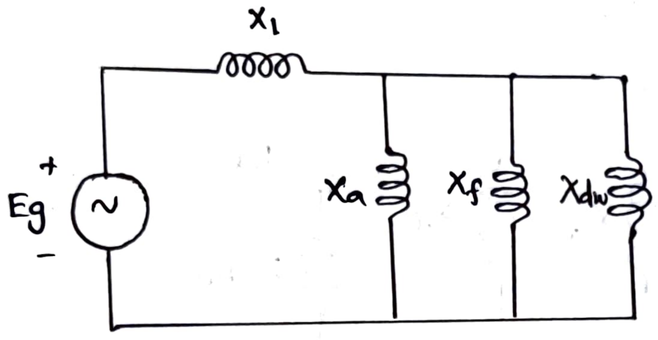
1) open conductor Fault
2) Two open conductor fault.

Series faults may occure with one or two broken conductors which creates open circuit. The series faults are charactorised by increase in voltage and fall in current in the faulted phase.

Short Circuit of A Synchronous Machine on no load:

Under short circuit condition, $R \ll X$. Thus the stator current lags the driving voltage by 90° and the armature reaction mmf is centered almost on the direct axis. Therefore the effective reactance of the machine may be assumed only along the direct axis.

Subtransient Reactance :-



During initial part of the short circuit, the damper and field windings have transformer secondary currents induced in them whose primary is the armature winding.

Subtansient reactance $X''_d = X_l + \left[\frac{1}{X_a} + \frac{1}{X_f} + \frac{1}{X_{dw}}\right]^{-1}$

X_a → Armature reaction reactance

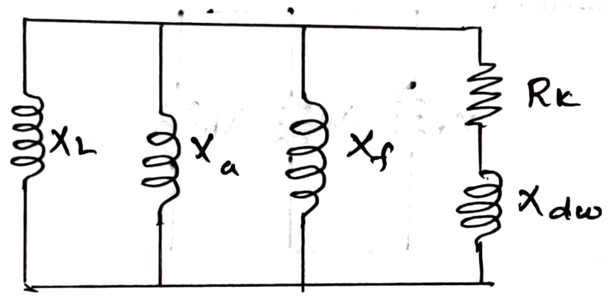
X_f → Field winding reactance

X_{dw} → Damper winding reactance.

The reactance represented by the machine in the initial period of the short circuit is called as the direct axis short circuit subtransient reactance of the machine.

If the damper winding resistance R_{dw} is inserted, the circuit time constant known as the direct axis short circuit subtransient time constant.

Thevenin Reactance :-



Thevenin's Reactance at the terminals of $R_K = X_{dw} + \left[\frac{1}{X_\ell} + \frac{1}{X_a} + \frac{1}{X_f}\right]^{-1}$

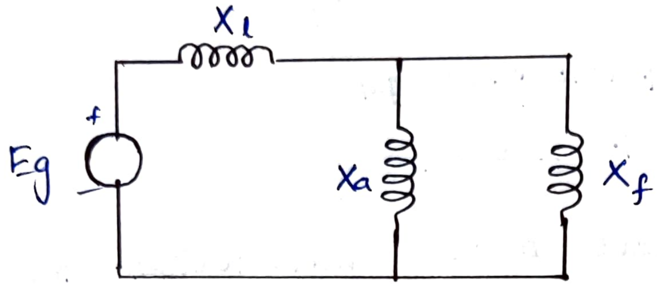
Direct axis short circuit subtransient time constant $\Big\}$ $T_d'' = \dfrac{\text{Thevenin Reactance}}{R_{dw}}$

$$T_d'' = \frac{X_{dw} + \left[\frac{1}{X_\ell} + \frac{1}{X_a} + \frac{1}{X_f}\right]^{-1}}{R_{dw}}.$$

For two pole alternator, X_d'' range from 0.07 to 0.12 pu. For water wheel alternator 0.1 to 0.35 p.u.

Transient Reactance :-

The damper circuit high resistance and $T_d'' \approx 0.35$ sec. Thus this component of current decays quickly. Therefore neglecting damper winding in equivalent circuit.

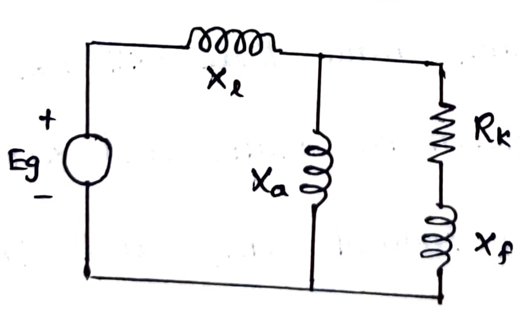


The equivalent reactance known as the direct axis short circuit transient reactance. It is the ratio of the induced emf on No load to the transient symmetrical rms current.

$$X_d' = X_l + \left[\frac{1}{X_a} + \frac{1}{X_f}\right]^{-1}$$

If the field winding resistance is inserted in the circuit, the circuit time constant known as direct axis short circuit time constant

Thevenin's Equivalent Circuit :-



Thevenin's reactance at the terminals of R_k $= X_f + \left[\frac{1}{X_l} + \frac{1}{X_a}\right]^{-1}$

Direct axis short circuit transient time constant } $T_d' = \dfrac{\text{Thevenin's reactance}}{R_f}$

$$T_d' = \frac{X_f + \left[\frac{1}{X_L} + \frac{1}{X_a}\right]^{-1}}{R_f}$$

Value of $X_d' = 0.1$ pu to 0.25 pu

$T_d' = 1$ to 2 sec.

The field time constant which characterises the decay of transient with the armature open circuited is called direct axis open circuit transient time constant.

$$T_{do}' = \frac{X_f}{R_f}$$

$$T_{do}' = 5 \text{ sec.}$$

$$T_d' = \frac{X_d'}{X_d} \times T_{do}'$$

Synchronous Reactance:-

When the disturbance is over, there will not be hunting of the rotor and hence there will not be any transformer action between the stator and rotor.

It is the ratio of induced emf and steady state rms current. It is the sum of leakage reactance and the armature reaction reactance.

During steady state period, the armature reaction produces the demagnetizing flux.

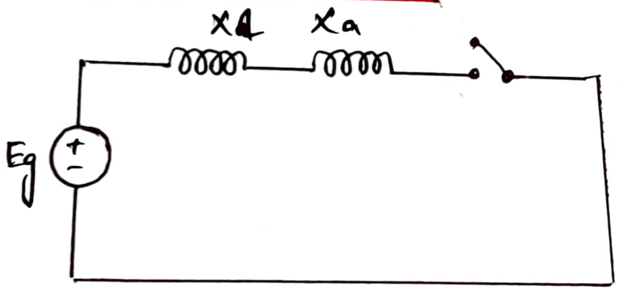
Synchronous reactance $X_d = X_a + X_l$

$X_a \rightarrow$ armature reaction reactance

$X_l \rightarrow$ leakage reactance.

$$X_d = \frac{|E_g|}{|I_1|}$$

<u>Steady state circuit Model:</u>



The machine offers a time varying reactance which changes from X_d'' to X_d' and X_d' ~~and~~ to X_d.

Subtransient current $|I''| = |E_g| / X_d''$

Transient current $|I'| = |E_g| / X_d'$

Steady state current $|I| = |E_g| / X_d$.

Bus Impedence Matrix

$$V_{bus} = Z_{bus} \cdot I_{bus}$$

$$Z_{bus} = [Y_{bus}]^{-1}$$

$$Z_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & & \vdots \\ Z_{n1} & Z_{n2} & & Z_{nn} \end{bmatrix}$$

* The diagonal elements are short circuit driving point impedences and the off diagonal elements are short circuit transfer impedences.
* Z_{bus} is symmetric then Y_{bus} is symmetric.
* Z_{bus} is a full matrix, In Y-bus some of the elements are zeros but in Z_{bus}, zero elements of Y_{bus} becomes non-zero elements.

Building Algorithm for Bus Impedence Matrix :-

Advantages :-

* Any modification of the network doesnot require complete rebuilding for Z_{bus}.
* Easily Computerized.

Assume original Z-bus with 'n' nodes. It is proposed to add new elements, one at a time to this network and get the modified $Z_{bus}^{(m)}$ matrix in the four ways.

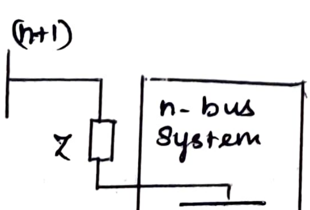
Modification 01: Add an element with impedence Z, connected between the reference node and a new node $(n+1)$.

Modification 02: Add an element, connected between an existing node i and a new node $n+1$

Modification 03: Add an element, connected between an existing node i and the reference node.

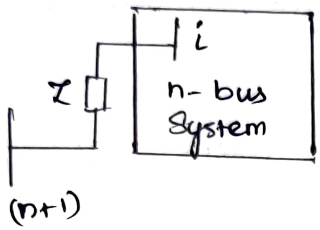
Modification 04: Add an element connected between existing nodes i and j.

Rule 01: Add an element with impedence Z, connected between the reference node and a new node $(n+1)$.



$$Z_{bus}^{new} = \left[\begin{array}{c|c} Z_{bus}^{old} & 0 \\ \hline 0 & Z \end{array}\right]$$

Rule no 02 : Add an element with impedence Z connected between an existing node i and a new node $(n+1)$.

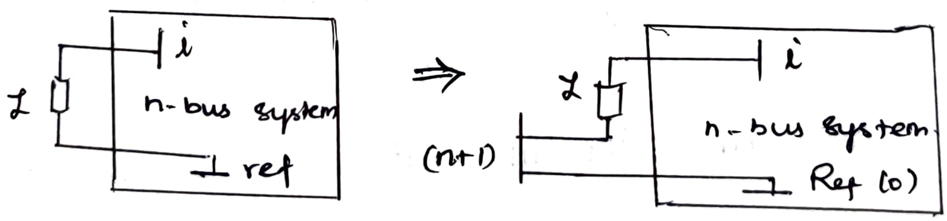


$$Z_{bus}^{new} = \begin{bmatrix} Z_{bus}^{old} & Z_i \\ Z_i^T & Z+Z_{ii} \end{bmatrix}$$

$Z_i \Rightarrow$ ith column of Z_{bus}.

$$\text{then, } Z_{bus}^{new} = \begin{bmatrix} Z_{bus}^{old} & \begin{matrix} Z_{1i} \\ Z_{2i} \\ \vdots \\ Z_{ni} \end{matrix} \\ Z_{i1}\ Z_{i2}, Z_{i3} \cdots Z_{in} & Z+Z_{ii} \end{bmatrix}$$

Rule no 03: Add an element with impedence Z, Connected between an existing node i and the reference node.

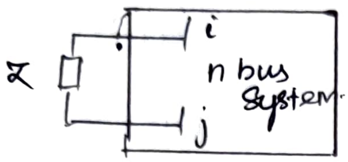


$$Z_{jk\,bus}^{m} = \frac{Z_{jk} - Z_{j(n+1)} \times Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

$$j, k = 1, 2, \ldots n$$

Here size of the network will not change because no new node added.

Rule no 04: Add an element with impedence z, connected between existing nodes i and j.



$$Z_{bus}^{new} = \begin{bmatrix} Z_{bus}^{old} & \text{Column } i - \text{Column } j \\ \text{Row } j - \text{Row } i & z + Z_{ii} + Z_{jj} - 2Z_{ij} \end{bmatrix}$$

$$Z_{jk\,bus}^{new} = \frac{Z_{jk} - Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

$$j, k = 1, 2 \ldots n$$

Here size of the matrix will not change because no new node added.

Circuit Breaker Selection Based on Momentry and Interrupting Duties:

Circuit breaker is a device used to isolate the faulty section from the healthy section during the fault.

Circuit interruption under fault condition is accomplished by circuit breakers. Circuit breaker can can be operated by manually or automatically.

The current interruption is achieved by movable contacts, when rapidly drawing an arc by means of arc extinction.

Momentry Current :-

The momentry current is computed using the Subtransient reactances of the utility sources (neighbouring systems), the synchronous generators, synchronous motors and induction motors.

Interrupting Fault current :-

The interrupting fault current is used to decide interrupting Capacity of the circuit breakers. This current is computed using subtransient reactance for generators and transient reactance for synchronous motors and Induction motor.

Types of rotating machine	First cycle	Interrupting cycle
Generator	$1.0\ X_d''$	$1.5\ X_d'$
Synchronous motor	$1.0\ X_d''$	$1.5\ X_d'$
Induction motor.		
above 1000 HP at 1800 rpm or less	$1.0\ X_d''$	$1.5\ X_d'$
above 250 HP at 3600 rpm	$1.0\ X_d''$	$1.5\ X_d'$
all the other 50 HP and above	$1.2\ X_d''$	$3.0\ X_d'$
Less than 50 HP	Neglect	Neglect.

Symmetrical Short Circuit:

Symmetrical fault is defined as the simultaneous Short circuit across all three phases. It occurs infrequently But it is the most severe fault. The other two phases carry identical currents except for the phase shift.

The reactance of the short circuit condition for the generator is time varying quantity,

X_d'' → Subtransient reactance for the first few cycles.

X_d' → Transient reactance for the next 30 cycles.

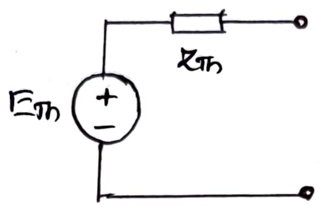
X_d° → Steady state synchronous reactance.

The Subtransient reactance (X_d'') is used for determining the interrupting capacity of the circuit breaker. The transient reactance (X_d') is used for relay setting and coordination, transient stability study.

The post fault voltages and currents in the network are obtained by superimposing these changes on the pre fault voltages and currents..

Thevenin's Theorem and Applications:

The changes that take place in the network voltages and currents due to the addition of an impedence between two network nodes are identical with those voltages and currents that would be caused by an emf placed in series with the impedence and having a magnitude and polarity equal to the prefault voltage that existed between the nodes and all other sources being zeroed.

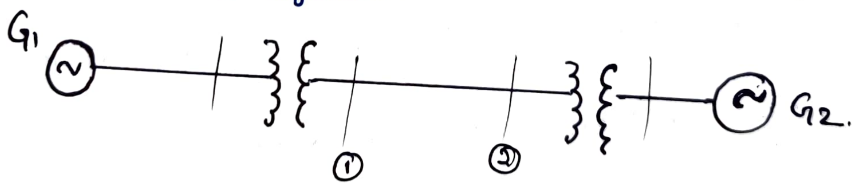


Applications:-

* The fault current can be evaluated
* The bus voltages and line currents during the fault can be determined.
* Post fault voltages and currents can be obtained by using prefault voltages and currents.

Short Circuit Analysis of Two Bus System

Consider two bus system.



In power system, loads are specified and the load currents are unknown.

The effect of load currents in the fault analysis are to express the loads by a constant impedence evaluated at the prefault bus voltages.

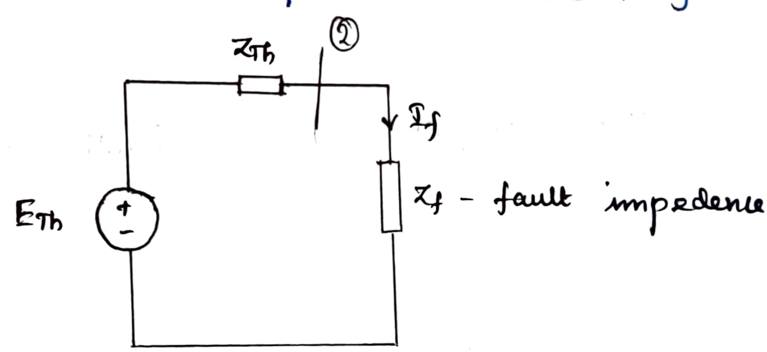
* Prefault bus voltages are obtained from results of the power flow solution.

* Loads are expressed by constant admittance using the prefault bus voltages.

* Replace reactances of synchronous machines by their subtransient / transient values.

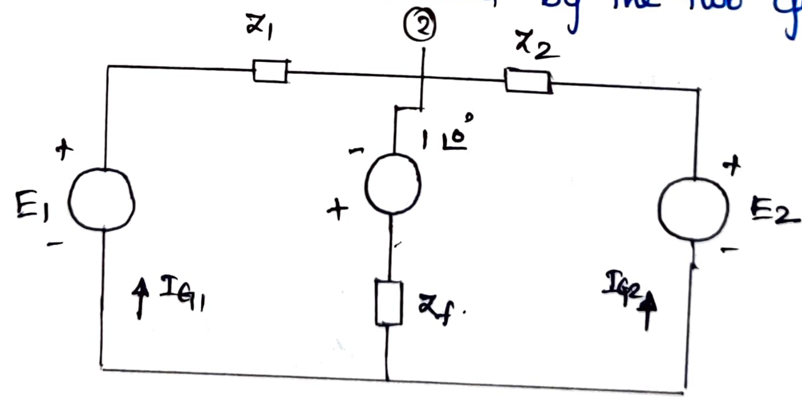
* Draw reactance diagram for the short circuit.

* Draw thevenin's equivalent viewed from the faulted bus.



$$\text{Fault current} = \frac{E_{Th}}{Z_{Th} + Z_f}$$

* Determine current contributed by the two generators.



$$I_{G_1} = I_f \times \frac{Z_2}{Z_1 + Z_2} \quad ; \quad I_{G_2} = I_f \times \frac{Z_1}{Z_1 + Z_2}$$

* Determine post fault bus voltages using

$$V_1^f = V_1^0 + \Delta V = V^0 + (-Z_{12} \times I_{G1})$$

↳ prefault voltage

$$V_2^f = V_2^0 + \Delta V = V_2^0 + Z_f I_f - 1 \angle 0^\circ$$

* Determine post fault line flows,

$$I_{ij} = \frac{V_i - V_j}{Z_{ij\ series}}.$$

V_i, V_j are bus voltages.

Z_{ij} series → series impedence between buses i & j.

Short Circuit Capacity (SCC) or Fault Level or Fault MVA

* It is defined as the product of the magnitudes of the prefault bus voltage and the post fault current.
* It is used for determining the dimension of a bus bar and the interrupting capacity of the circuit breaker.

From thevenin's equivalent circuit,

$$\text{Fault current} = \frac{E_{Th}}{Z_{Th}}$$

$$|I_f| = \frac{E_{Th}}{X_{Th}} \quad \text{p.u}$$

E_{Th} → prefault voltage.

$$\text{Base current} = \frac{MVA_b}{\sqrt{3} \times kV_b} \times 10^3$$

Fault in KA = I_f in p.u × Base current

$$I_f = \frac{E_{Th}}{X_{Th}} \times \frac{MVA_b}{\sqrt{3}\, KV_b} \times 10^3$$

Short circuit Capacity $SCC = |E_{Th}| \times |I_f|$

$$= |E_{Th}| \times \frac{|E_{Th}|}{X_{Th}}$$

$$= \frac{|E_{Th}|^2}{X_{Th}} \text{ p.u MVA.}$$

Prefault voltage $\approx 1\angle 0^\circ$

$$\therefore SCC = \frac{1}{X_{Th}} \text{ p.u MVA.}$$

$$SCC = \frac{1}{X_{Th}} \times MVA_b \text{ MVA}$$

$MVA_b \rightarrow$ Base MVA.

The SCC have a tendency to grow as new generators are added and additional lines are built. The SCC is reduced by introducing artificial series reactors.

Systematic Short Circuit Computation (Z bus in Phase Frame)

Fault Analysis Using Z-bus Matrix:

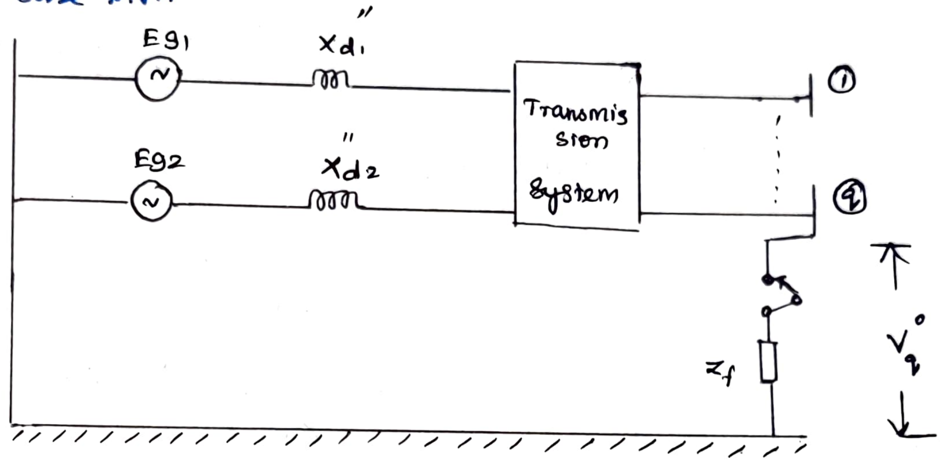
Consider typical bus of n-bus system. The system is assumed to be operating under balanced condition and a per phase model is used.

Step 1 : Draw the prefault per phase network :-

(positive sequence network)

Each machine is represented by a constant voltage source behind proper reactance (X_d'', X_d' or X_d).

Transmission line reactance are expressed in per unit on a common base MVA.



Let us assume 3 φ fault occurs at bus q through a fault impedence Z_f.

Prefault Bus Voltages :-

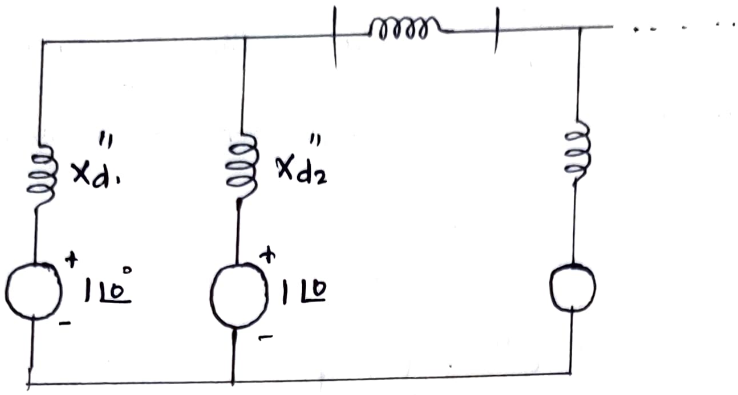
It is obtained from the power flow solution

$$\text{Initial bus voltages, } V_{bus}^{0} = \begin{bmatrix} V_1^0 \\ \vdots \\ V_q^0 \\ \vdots \\ V_N^0 \end{bmatrix}$$

A good approximation to represent the load by a constant impedence evaluated at the prefault voltage

$$\text{ie } Z_{iL} = \frac{|V_i^0|^2}{S_L^*}$$

Step 2 : Obtain Z_{bus} matrix bus building Algorithm.

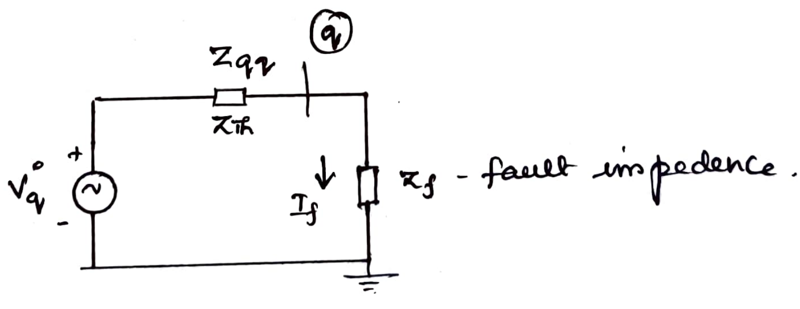


Assume one node as reference node and short circuiting all the voltage sources, Determine the Z_{bus} using step by step bus building algorithm.

Step 3 : Obtain the fault current

Let us assume the prefault currents are negligible.

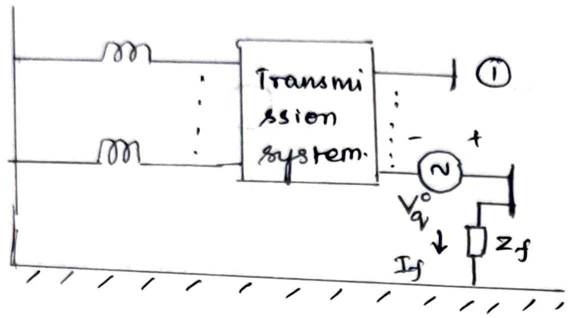
The changes in the network voltage caused by the added voltage V_q^0 with all other sources. short circuited. By representing all components and loads by their appropriate impedences.



$$\text{Fault current} = \frac{V_q^0}{Z_{qq} + Z_f}$$

where Z_{qq} is the diagonal element of the Z_{bus} matrix.

Step 4 : Obtain the Thevenin's network by inserting the Thevenin's voltage source V_q^0 in series with Z_f and compute change in bus voltages using network equation.



The current entering every bus is zero except at the faulted bus.

$$I_1 = I_2 \cdots = I_N = 0 \text{ except } I_q$$

$$I_q = -I_f$$

$$I_{bus} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_q \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_f \\ \vdots \\ 0 \end{bmatrix}$$

$$I_{bus\,(F)} = Y_{bus} \cdot \Delta V_{bus}$$

Change in bus voltage

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_q \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1q} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2q} & \cdots & Z_{2N} \\ \vdots & & & & & \\ Z_{q1} & Z_{q2} & \cdots & Z_{qq} & \cdots & Z_{qN} \\ \vdots & & & & & \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nq} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_f \\ \vdots \\ 0 \end{bmatrix}$$

Step 5 : Post Fault Bus Voltages :-

Post fault bus voltages are obtained by superposition of the prefault bus voltages and the change in the bus voltages.

$$V_{bus(f)} = V_{bus}^{0} + \Delta V_{bus}.$$

$$\begin{bmatrix} V_1^f \\ \vdots \\ V_q^f \\ \vdots \\ V_N^f \end{bmatrix} = \begin{bmatrix} V_1^0 \\ \vdots \\ V_q^0 \\ \vdots \\ V_N^0 \end{bmatrix} + \begin{bmatrix} Z_{11} & \cdots & Z_{1q} & \cdots & Z_{1N} \\ \vdots & & \vdots & & \vdots \\ Z_{q1} & \cdots & Z_{qq} & \cdots & Z_{qN} \\ \vdots & & & & \vdots \\ Z_{N1} & \cdots & Z_{Nq} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ -I_f \\ \vdots \\ 0 \end{bmatrix}$$

$$V_1^f = V_1^0 - Z_{1q} I_f$$

$$V_2^f = V_2^0 - Z_{2q} I_f$$

$$\vdots$$

$$V_q^f = V_q^0 - Z_{qq} I_f$$

$$\vdots$$

$$V_N^f = V_N^0 - Z_{Nq} I_f$$

$\therefore$ In general, $V_i^f = V_1^0 - Z_{iq} I_f$.

Bus voltages during the fault :-

Substitude in the fault current

$$V_i^f = V_1^0 - \frac{Z_{iq} \times V_q^0}{Z_{qq} + Z_f} \quad ; \quad i \neq q$$

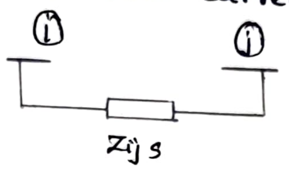
$$V_q^f = \frac{Z_f}{Z_{qq} + Z_f} \times V_q^0 \quad ; \quad i = q$$

If the short circuit is solid or bolted ($Z_f = 0$)

$$\text{Fault current } I_f = \frac{V_q^0}{Z_{qq}}$$

$$V_q^f = 0 \quad , \quad V_i^f = V_i^0 - \frac{Z_{iq}}{Z_{qq}} \times V_q^0 \; ; \; i \neq q$$

Step 6 : Post Fault Line currents :



Let us consider the line connecting between buses i and j with series impedence $Z_{ij\,s}$

$$\text{Post fault line current } I_{ij}^f = \frac{V_i^f - V_j^f}{Z_{ij \text{ series}}}$$

From the bus impedence matrix, the fault current and bus Voltages during the fault and post fault line currents are obtained for any faulted bus.

UNSYMMETRICAL FAULT ANALYSIS

When the network is unsymmetrically faulted or loaded, neither the phase sequence currents nor the phase voltages will possess three phase symmetry.

If unsymmetrical fault occurs, the unbalanced currents will flow in the system. we are using symmetrical components to analyze symmetrical faults.

Types of unsymmetrical fault :-

* Line to ground fault (L-G)
* Line to Line fault (L-L)
* Double line to Ground fault (L-L-G)
* open conductor fault.

Causes of unsymmetrical Fault :-

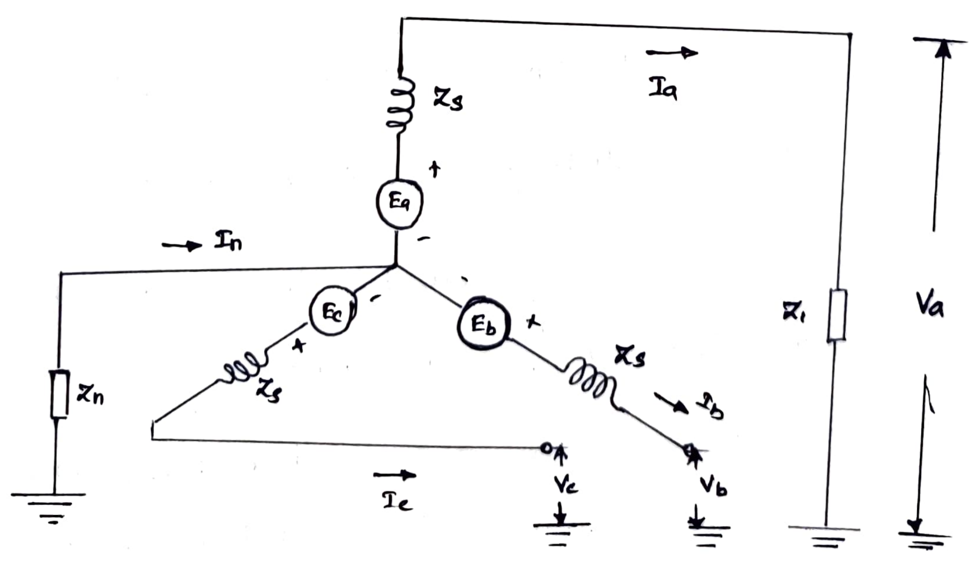
* Lightning, wind damage, trees falling across lines
* ~~[illegible]~~ Vehicles colliding with towers or poles
* Birds shorting lines, breaks due to excessive ice loading or snow loading, salt spray.
* Some other causes are due to breaking one or two conductors or the action of fuses and other protective devices that may not open the three phase simultaneously.

Short circuit Analysis of Unbalanced low order systems.

* Draw the positive, negative and zero sequence networks with their appropriate description.
* Choose of type of fault (L-G, L-L-G, L-L) and location of fault and Mathematical description for the particular type of fault.
* Using thevenin's theorem or bus impedence matrix, determine the solution of the network equations. Fault current, post fault current, pre fault current & voltages are found as the point of fault, all the bus voltages and line flows.

Single Line to Ground Fault (L-G Fault):-

The single line to ground fault, the most common type is caused by lightning or conductors making contact with ground structure.



suppose a line to ground fault on phase 'a' connected to ground through impedence Z_f. Assume the generator is initially on no load, the conductors at the fault bus 'K' are expressed as

$$\left.\begin{aligned} V_a &= Z_f I_a \\ I_b &= I_c = 0 \\ I_f &= I_a \end{aligned}\right\} \quad \text{---} \; (1)$$

symmetrical components of currents are,

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad \text{----} \; (2)$$

Substitude $I_b = I_c = 0$, the symmetrical components of currents are

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \quad \text{-----} \; (3)$$

From the above matrix, we can get

$$I_a^0 = I_a/3$$

$$I_a^+ = I_a/3$$

$$I_a^- = I_a/3 = I_f/3$$

$$\therefore \; I_a^+ = I_a^- = I_a^0 = \frac{I_a}{3} = \frac{I_f}{3} \quad \text{------} \; (4)$$

From sequence networks of the generator, the symmetrical voltages are given by

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{kk}^0 & 0 & 0 \\ 0 & Z_{kk}^+ & 0 \\ 0 & 0 & Z_{kk}^- \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$\left.\begin{aligned} V_a^0 &= -Z_{kk}^0 I_a^0 = -Z_{kk} I_a^+ \\ V_a^+ &= E_a - Z_{kk}^+ I_a^+ \\ V_a^- &= -Z_{kk}^- I_a^- = -Z_{kk}^- I_a^+ \end{aligned}\right\} \quad \text{-----} \quad (5)$$

The phase voltages are given by

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} \quad \text{------} \quad (6)$$

From the above matrix, we can get

$$V_a = V_a^0 + V_a^+ + V_a^-$$

From condition, $V_a = Z_f I_a$

$$\therefore \quad V_a^0 + V_a^+ + V_a^- = Z_f I_a \quad \text{------} \quad (7)$$

Substitude symmetrical components of voltages from eqn (5), we get

$$-Z_{kk}^0 I_a^+ + E_a - Z_{kk}^+ I_a^+ + (-Z_{kk}^- I_a^+) = Z_f I_a$$

$$E_a - I_a^+ \left[Z_{kk}^0 + Z_{kk}^+ + Z_{kk}^- \right] = Z_f \times 3 I_a^+$$

$$I_a^+ \left[Z_{kk}^0 + Z_{kk}^+ + Z_{kk}^- + 3 Z_f \right] = E_a$$

$$I_a^+ = \frac{E_a}{Z_{kk}^0 + Z_{kk}^+ + Z_{kk}^- + 3 Z_f} \quad \text{----} \quad (8)$$

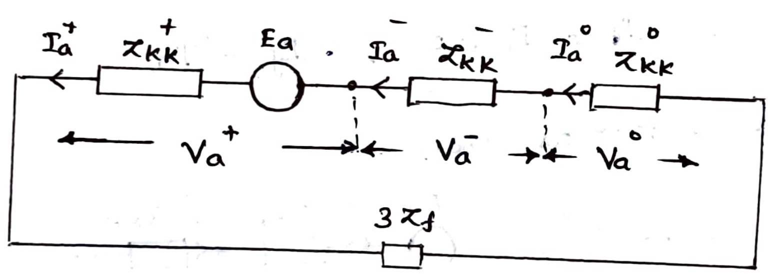
The fault current is,

$$I_f = I_a = 3 I_a^{+} = \frac{3 E_a}{Z_{kk}^{0} + Z_{kk}^{+} + Z_{kk}^{-} + 3 Z_f} \quad --- (9)$$

Similarly the symmetrical components of phase voltages and phase voltages at the fault point are obtained.

Sequence Network :-

The zero, positive and negative sequences are connected in series. Thus for LG faults, the thevenin's impedance to the fault point is obtained for each sequence network and are connected in series.



$Z_{kk}^{0} = Z_s + 3Z_n$

Mostly $Z_{kk}^{+} = Z_{kk}^{-}$

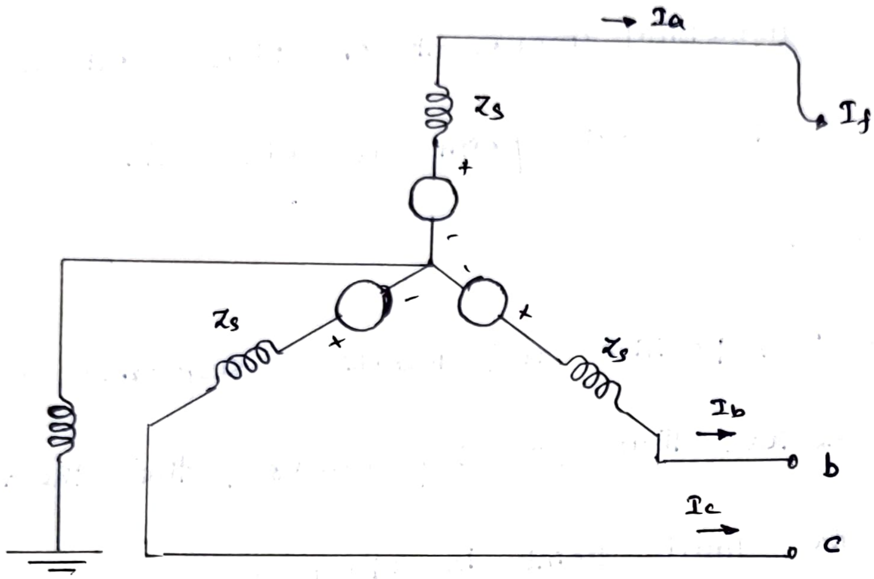
* If the generator is solidly grounded, $Z_n = 0$ and for bolted faults (or direct short circuit fault) or solid fault, $Z_f = 0$

* If the neutral of generator is ungrounded, the zero sequence network is open circuited.

$$\therefore I_a^{+} = I_a^{-} = I_a^{0} = 0 \quad \text{and} \quad I_f = 0$$

Direct Short Circuit (or) Bolted Fault :-

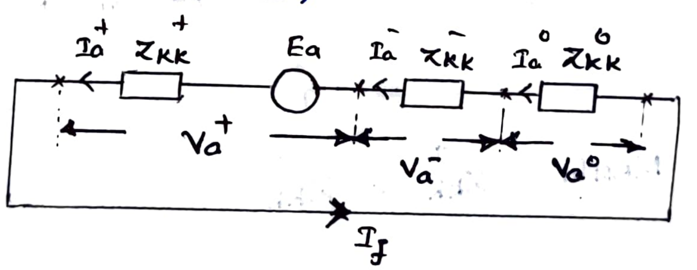
When the direct short circuit fault at phase 'a', the fault impedence $Z_f = 0$.



The conditions of the fault at bus k are

$V_a = 0$, $I_b = I_c = 0$, $I_f = I_a$

The sequence networks are,



$Z_{kk}^{0} = Z_s + 3Z_n$

$$I_a^{+} = I_a^{-} = I_a^{0} = I_f \quad \text{------} \quad (10)$$

$$I_f = \frac{E_a}{Z_{kk}^{+} + Z_{kk}^{-} + Z_{kk}^{0}} \quad \text{-----} \quad (11)$$

Prefault sequence voltages :-

Since the fault is assumed to occure when the prefault system is under balanced condition, all prefault voltages contain only positive sequence components.

$$V_{os} = \begin{bmatrix} V_1^+ \\ V_1^- \\ V_1^0 \\ \vdots \\ V_q^+ \\ V_q^- \\ V_q^0 \\ \vdots \\ V_n^+ \\ V_n^- \\ V_n^0 \end{bmatrix} = \begin{bmatrix} V_{10}^+ \\ 0 \\ 0 \\ \vdots \\ V_{q0}^+ \\ 0 \\ 0 \\ \vdots \\ V_{n0}^+ \\ 0 \\ 0 \end{bmatrix} \quad ---- (12)$$

Post Fault voltages:-

The post positive sequence bus voltages are

$$V_f^+ = V_0^+ + Z_{ik}^+ I_f \quad ---- (13)$$

Since the fault is injected at bus k,

$$I_f = \begin{bmatrix} 0 \\ 0 \\ I_f^+ \\ 0 \\ 0 \end{bmatrix} \quad ------- (14)$$

$$\therefore V_f^+ = V_0^+ - Z_{ik} I_f^+ \quad --- (15)$$

$$\therefore \left.\begin{aligned} V_{f1}^+ &= V_0^+ - Z_{1k}^+ I_f^+ \\ &\vdots \\ V_{fk}^+ &= V_0^+ - Z_{kk}^+ I_f^+ \\ &\vdots \\ V_{fn} &= V_0^+ - Z_{nk}^+ I_f^+ \end{aligned}\right\} \text{Positive sequence voltages} \quad ----- (16)$$

The post fault negative sequence bus voltages are

$$\left.\begin{aligned} V_{f1}^{-} &= -Z_{1k}^{-} I_f^{-} \\ &\vdots \\ V_{fk}^{-} &= -Z_{kk}^{-} I_f^{-} \\ &\vdots \\ V_{nk} &= -Z_{nk}^{-} I_f \end{aligned}\right\} \quad \text{- - - -} \quad (19)$$

The post fault zero sequence bus voltages are

$$\begin{aligned} V_{f1}^{0} &= -Z_{1k}^{0} I_f^{0} \\ &\vdots \\ V_{fk}^{0} &= -Z_{kk}^{0} I_f^{0} \\ &\vdots \\ V_{fn}^{0} &= -Z_{nk}^{0} I_f^{0} \end{aligned}$$

Sequence Line currents :-

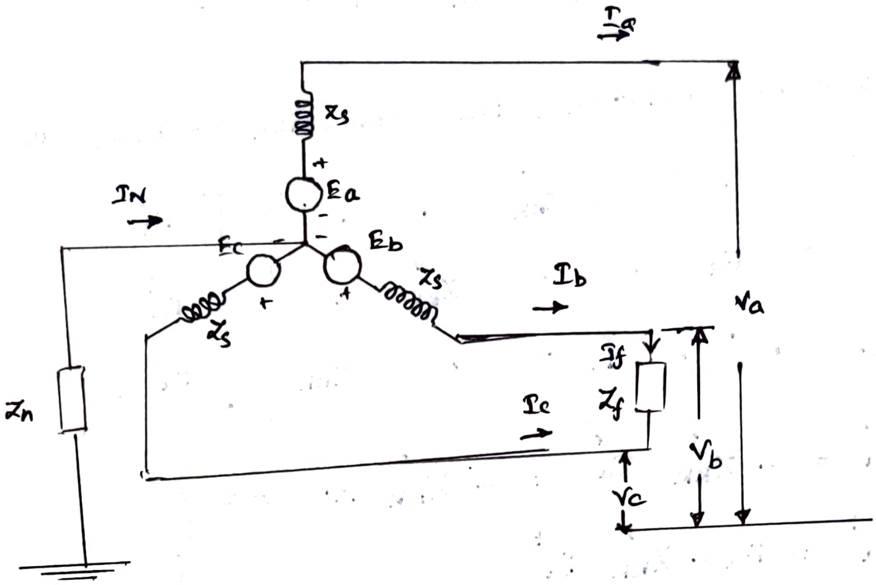
Positive sequence current $I_{ij}^{+} = \dfrac{V_{fi}^{+} - V_{fj}^{+}}{Z_{ij}^{+}}$

Negative sequence current $I_{ij}^{-} = \dfrac{V_{fi}^{-} - V_{fj}^{-}}{Z_{ij}^{-}}$

Zero sequence current $I_{ij}^{0} = \dfrac{V_{fi}^{0} - V_{fj}^{0}}{Z_{ij}^{0}}$

Line to Line Fault:

Consider a three phase generator with a fault through an impedence Z_f between phases b and c. Assume generator is unloaded, condition at the fault bus k are expressed by the following reason.



$$\left.\begin{aligned} &I_b = -I_c , \quad I_a = 0 \text{ (unloaded generator)}, \quad V_b - V_c = Z_f I_b \\ &\therefore V_c = V_b - Z_f I_b \end{aligned}\right\} \quad \text{—(1)}$$

Substitude for $I_b = -I_c$, $I_a = 0$, the symmetrical components of currents are

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$I_a^0 = \frac{1}{3}\left[0 + I_b - I_b\right] = 0$$

$$I_a^+ = \frac{1}{3}\left[a I_b - a^2 I_b\right]$$

$$I_a^- = \frac{1}{3}\left[a^2 I_b - a I_b\right]$$

$$\therefore \quad I_a^+ = -I_a^- \quad \text{and} \quad I_a = 0 \quad \text{-----} \ (2)$$

From sequence network, the symmetrical voltages are given by

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{kk}^0 & 0 & 0 \\ 0 & Z_{kk}^+ & 0 \\ 0 & 0 & Z_{kk}^- \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$\left.\begin{aligned} V_a^0 &= -Z_{kk}^0 I_a^0 = -Z_{kk}^0 \times 0 = 0 \\ V_a^+ &= E_a - Z_{kk}^+ I_a^+ \\ V_a^- &= -Z_{kk}^- I_a^- = Z_{kk}^+ I_a^+ \end{aligned}\right\} \text{----} \ (3)$$

The phase currents are given by.

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_a^+ \\ -I_a^+ \end{bmatrix}$$

$$\left.\begin{aligned} I_a = 0, \quad I_b &= a^2 I_a^+ - a I_a^+ = I_a^+ (a^2 - a) \\ I_c &= a I_a^+ - a^2 I_a^+ = I_a^+ (a - a^2) \end{aligned}\right\} \text{---} \ (4)$$

The voltages throughout the zero sequence network must be zero since there are no zero sequence sources because $I_a^0 = 0$, current is not being injected into that network due to the fault. Hence LL fault calculation donot involve zero sequence network.

The phase voltages are

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

$V_a^0 = 0$,
$$\left.\begin{aligned} V_a &= V_a^+ + V_a^- \\ V_b &= a^2 V_a^+ + a V_a^- \\ V_c &= a V_a^+ + a^2 V_a^- \end{aligned}\right\} \quad \text{----- (5)}$$

From the condition, $V_b - V_c = Z_f I_b$, then

$$a^2 V_a^+ + a V_a^- - a V_a^+ - a^2 V_a^- = Z_f I_b$$

$$V_a^+ (a^2 - a) - V_a^- (a^2 - a) = Z_f I_b$$

$$(a^2 - a)(V_a^+ - V_a^-) = Z_f I_b \quad \text{----- (6)}$$

We know that $I_b = (a^2 - a) I_a^+$, then

$$\cancel{(a^2 - a)} (V_a^+ - V_a^-) = Z_f [\cancel{(a^2 - a)} I_a^+]$$

$$V_a^+ - V_a^- = Z_f I_a^+ \quad \text{----- (7)}$$

substitude V_a^+, V_a^- , and we get

$$E_a - Z_{KK}^+ I_a^+ - [- Z_{KK}^- I_a^-] = Z_f I_a^+$$

$$E_a - (Z_{KK}^+ + Z_{KK}^-) I_a^+ = Z_f I_a^+$$

$$E_a = I_a^+ [Z_{KK}^+ + Z_{KK}^- + Z_f]$$

$$I_a^+ = \frac{E_a}{Z_{KK}^+ + Z_{KK}^- + Z_f} \quad \text{----- (8)}$$

$$I_a^- = -I_a^+$$

$$I_a^0 = 0$$

current in phase domain,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \begin{bmatrix} 0 + I_a^+ - I_a^+ \\ 0 + (a^2-a) I_a^+ \\ 0 + (a-a^2) I_a^+ \end{bmatrix} = \begin{bmatrix} 0 \\ (a^2-a) I_a^+ \\ (a-a^2) I_a^+ \end{bmatrix}$$

The fault current is $I_b = -I_c = (a^2-a) I_a^+$

$$I_b = (-0.5 - j0.866 + 0.5 - j0.866) I_a^+$$

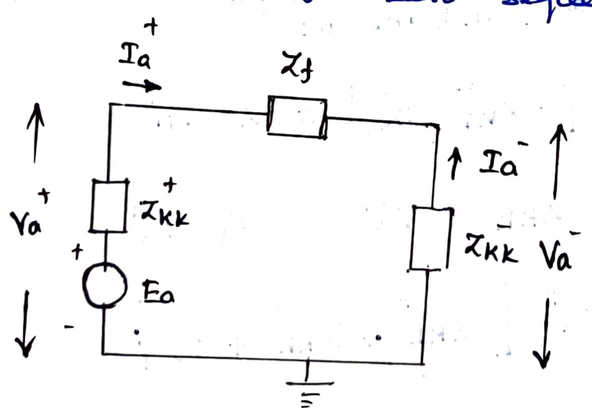
$$= -j1.732 \, I_a^+$$

$$I_b = -j\sqrt{3} \, I_a^+$$

Substitude this value in I_a^+

$$I_f = I_b = \frac{-j\sqrt{3} \, E_a}{Z_{kk}^+ + Z_{kk}^- + Z_f} \quad \text{----- (9)}$$

Now possitive sequence network is connected in parallel with the negative sequence network through the fault impedence Z_f and no connection for zero sequence network because $V_a^0 = 0$



in Practical

$$Z_{kk}^+ = Z_{kk}^-$$

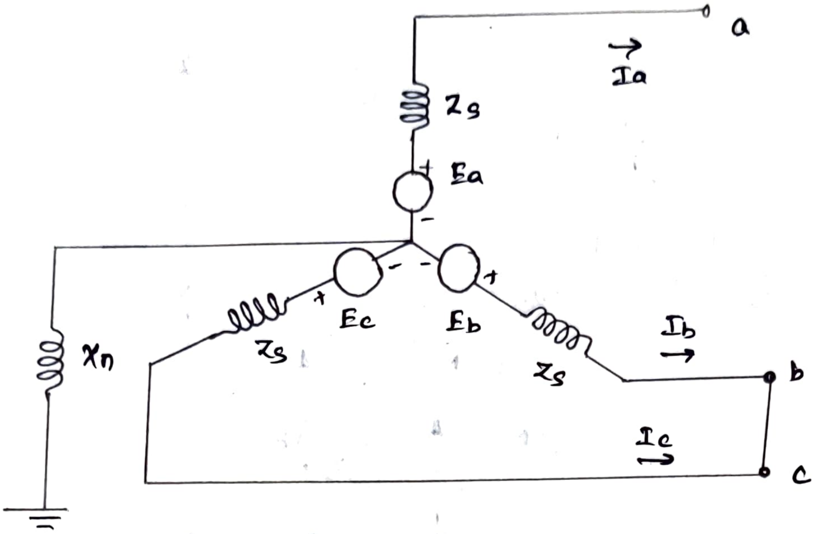
From the network, we know that

$$I_a^- = -I_a^+, \quad I_a^0 = 0, \quad I_a^+ = \frac{E_a}{Z_{kk}^+ + Z_{kk}^- + Z_f} \quad \text{-- (10)}$$

$$V_a^+ = V_a^- + Z_f I_a^+$$

$$I_f = I_b = I_a^+ (a^2-a) = -j\sqrt{3} \, I_a^+ \quad \text{------ (11)}$$

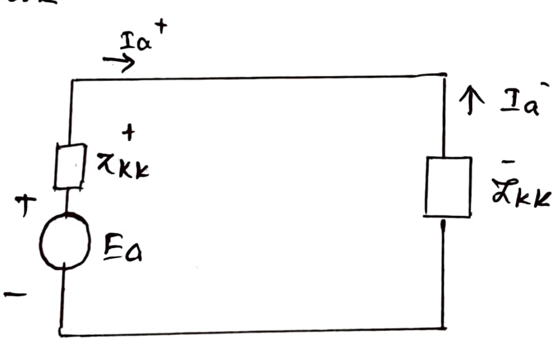
Direct short circuit or Bolted L-L Fault :-



Fault impedence $Z_f = 0$, The conditions of the fault at k bus are,

$$I_a = 0 \quad , \quad I_b = -I_c \;, \; V_b = V_c \qquad \text{——} \; (1)$$

Sequence network

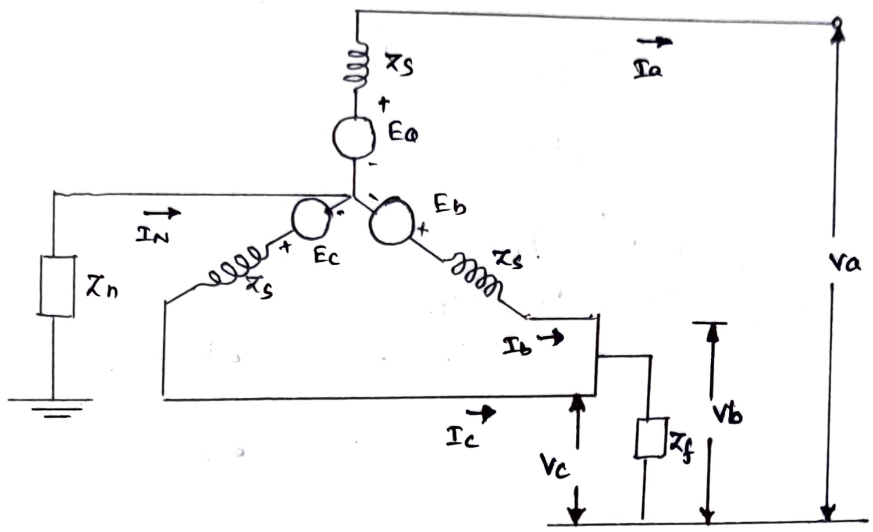


$$I_a^+ = -I_a^- \quad , \quad I_f = \frac{E_a(a^2 - a)}{Z_{kk}^+ + Z_{kk}^-} = \frac{-j\sqrt{3}\,E_a}{Z_{kk}^+ + Z_{kk}^-} \qquad \text{——} \; (2).$$

Double Line to Ground Fault :

A three phase generator with a fault on phases b and c through an impedence Z_f to ground. Assume the generator is initially on no-load, the conditions at the fault k are expressed by the following relations.

$I_a = 0$, $I_b + I_c = I_f$, $V_b = V_c = Z_f I_f = Z_f (I_b + I_c)$ --- ①



The symmetrical components of voltages are

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \text{------ ②}$$

Substitude $V_b = V_c$

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix}$$

$$V_a^0 = \frac{1}{3}(V_a + V_b + V_b) = \frac{1}{3}(V_a + 2V_b)$$

$$V_a^+ = \frac{1}{3}(V_a + aV_b + a^2 V_b)$$

$$= \frac{1}{3}(V_a + V_b(a + a^2)) \qquad \because 1 + a + a^2 = 0$$

$$a + a^2 = -1$$

$$V_a^+ = \frac{1}{3}(V_a - V_b)$$

$$V_a^- = \frac{1}{3}(V_a + a^2 V_b + aV_b)$$

$$= \frac{1}{3}(V_a + V_b(a + a^2))$$

$$V_a^- = \frac{1}{3}(V_a - V_b)$$

$$\therefore \quad V_a^+ = V_a^- \quad \text{--- (3)}$$

The phase currents are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$I_a = I_a^0 + I_a^+ + I_a^-$$

$$I_b = I_a^0 + a^2 I_a^+ + a I_a^-$$

$$I_c = I_a^0 + a I_a^+ + a I_a^-$$

and $I_f = I_b + I_c = I_a^0 + a^2 I_a^+ + a I_a^- + I_a^0 + a I_a^+ + a^2 I_a^-$

$$= 2 I_a^0 + I_a^+ (a^2 + a) + I_a^- (a^2 + a)$$

$$= 2 I_a^0 + I_a^+ (-1) + I_a^- (-1)$$

$$= 2 I_a^0 - (I_a^+ + I_a^-) . \quad \text{--- (4)}$$

From the condition, $I_a = I_a^0 + I_a^+ + I_a^- = 0$

$$I_a^+ + I_a^- = -I_a^0 \quad \text{--- (5)}$$

and $I_b + I_c = 2 I_a^0 - (I_a^+ + I_a^-)$

$$= 2 I_a - (-I_a^0)$$

$$I_b + I_c = 3 I_a^0$$

and $V_b = Z_f (I_b + I_c)$

$$\therefore \quad V_b = Z_f \times 3 I_a^0 \quad \text{--- (6)}$$

The phase voltages are given by

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

$$V_a = V_a^0 + V_a^+ + V_a^-$$

$$V_b = V_a^0 + a^2 V_a^+ + a V_a^- = V_a^0 + a^2 V_a^+ + a V_a^+ \qquad \because V_a^+ = V_a^-$$

$$V_b = V_a^0 + V_a^+ (a + a^2) \qquad \because a^2 + a = -1$$

$$V_b = V_a^0 - V_a^+ \qquad \therefore [V_b = 3 Z_f I_a^0] \quad \text{---- (7)}$$

and $$V_a^0 - V_a^+ = 3 Z_f I_a^0 \quad \text{----- (8)}$$

The symmetrical components Voltages are given by

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{KK}^0 & 0 & 0 \\ 0 & Z_{KK}^+ & 0 \\ 0 & 0 & Z_{KK}^- \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$\left.\begin{aligned} V_a^0 &= -Z_{KK}^0 I_a^0 \\ V_a^+ &= E_a \times - Z_{KK}^+ I_a^+ \\ V_a^- &= -Z_{KK}^- I_a^- \end{aligned}\right\} \quad \text{----- (9)}$$

Substitude V_a^0, V_a^+ in eqn (8)

$$-Z_{KK}^0 I_a^0 - [E_a - Z_{KK}^+ I_a^+] = 3 Z_f I_a^0$$

$$-[E_a - Z_{KK}^+ I_a^+] = Z_{KK}^0 I_a^0 + 3 Z_f I_a^0$$

$$I_a^0 = \frac{-[E_a - Z_{KK}^+ I_a^+]}{Z_{KK}^0 + 3 Z_f} \quad \text{--- (10)}$$

and $V_a^+ = V_a^-$

$$\therefore E_a - Z_{KK}^+ I_a^+ = -Z_{KK}^- \cdot I_a^-$$

$$I_a^- = \frac{(E_a - Z_{KK}^+ I_a^+)}{Z_{KK}^-} \quad \text{--- (11)}$$

then, $-I_a^0 = I_a^+ + I_a^-$

$$I_a^+ = -\left[I_a^- + I_a^0\right] = -I_a^- - I_a^0$$

$$I_a^+ = \left[\frac{E_a - Z_{kk}^+ I_a^+}{Z_{kk}^-}\right] + \left[\frac{E_a - Z_{kk}^+ I_a^+}{Z_{kk}^0 + 3Z_f}\right]$$

$$= I_a \cdot \left[1 + \frac{Z_{kk}^+}{Z_{kk}^-} + \frac{Z_{kk}^+}{Z_{kk}^0 + 3Z_f}\right]$$

$$= \frac{E_a}{Z_{kk}^-} + \frac{E_a}{Z_{kk}^0 + 3Z_f}$$

$$I_a^+\left[Z_{kk}^-\left(Z_{kk}^0 + 3Z_f\right) + Z_{kk}^+\left(Z_{kk}^0 + 3Z_f\right) + Z_{kk}^+ Z_{kk}^-\right] = E_a\left[Z_{kk}^0 + 3Z_f + Z_{kk}^-\right]$$

$$I_a^+\left[Z_{kk}^+\left(Z_{kk}^0 + 3Z_f + Z_{kk}^-\right) + Z_{kk}^-\left(Z_{kk}^0 + 3Z_f\right)\right] = E_a\left(Z_{kk}^0 + 3Z_f + Z_{kk}^-\right)$$

$$I_a^+ = \frac{E_a\left(Z_{kk}^0 + 3Z_f + Z_{kk}^-\right)}{\left(Z_{kk}^0 + 3Z_f + Z_{kk}^-\right)\left[Z_{kk}^+ + \frac{Z_{kk}^-\left(Z_{kk}^0 + 3Z_f\right)}{\left(Z_{kk}^0 + 3Z_f + Z_{kk}^-\right)}\right]}$$

$$I_a^+ = \frac{E_a}{Z_{kk}^+ + \frac{Z_{kk}^-\left(Z_{kk}^0 + 3Z_f\right)}{Z_{kk}^0 + 3Z_f + Z_{kk}^-}} \quad \text{--- (12)}$$

The fault current $I_f = 3 I_a^0$

$$\therefore I_f = -3 \times \left[\frac{E_0 - Z_{kk}^+ I_a^+}{Z_{kk}^0 + 3Z_f}\right]$$

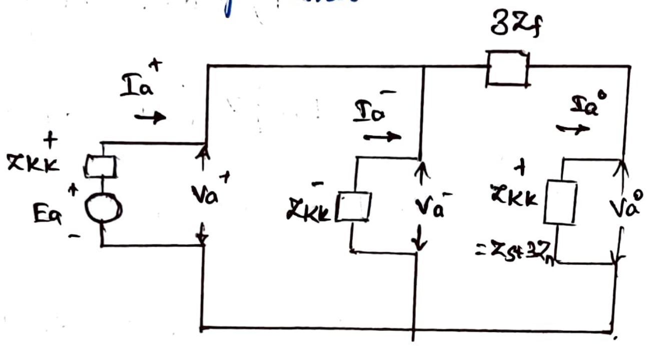
Substituting I_a^+,

$$I_f = \frac{-3}{Z_{kk}^{0} + 3Z_f}\left[E_a - \frac{Z_{kk}^{+}\,E_a}{Z_{kk}^{+} + \dfrac{Z_{kk}^{-}(Z_{kk}^{0} - 3Z_f)}{Z_{kk}^{0} + Z_{kk}^{-} + 3Z_f}}\right]$$

$$I_f = \frac{-3}{Z_{kk}^{0} + 3Z_f}\left[\frac{E_a \times Z_{kk}^{-}\,(Z_{kk}^{0} + 3Z_f)}{Z_{kk}^{+} \times Z_{kk}^{0} + 3Z_f Z_{kk}^{+} + Z_{kk}^{+} Z_{kk}^{-} + Z_{kk}^{-} Z_{kk}^{0} + 3Z_f Z_{kk}^{-}}\right] \quad \text{------ (13)}$$

Sequence Network :

The positive, negative and zero sequence networks are connected in parallel.



From sequence network,

$$V_a^{+} = V_a^{-} = V_a^{0} + 3Z_f I_a^{0}$$

$$I_a^{+} + I_a^{-} = -I_a^{0}$$

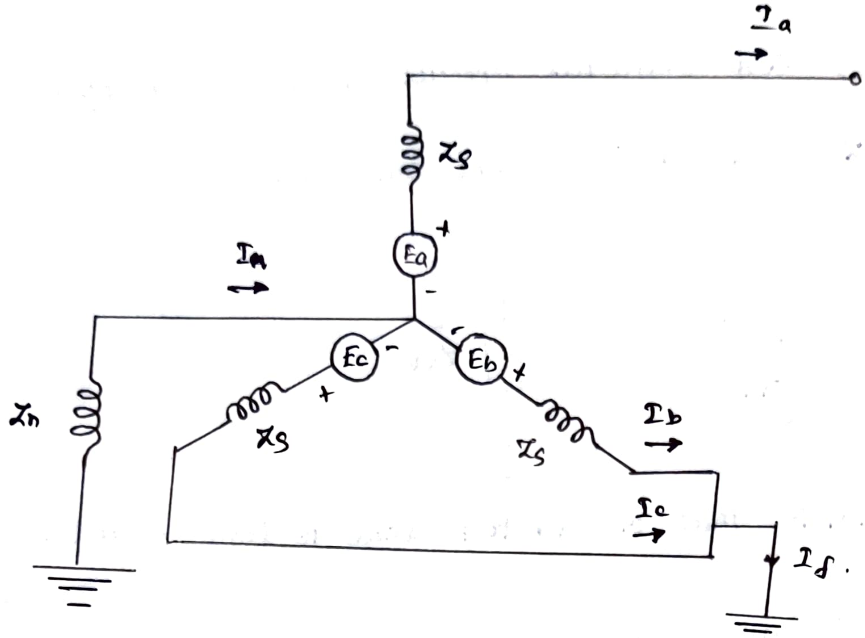
Total Z consisting of Z_{kk}^{+} in series with parallel combination of Z_{kk}^{-} and $Z_{kk}^{0} + 3Z_f$

$$I_a^{+} = \frac{E_a}{Z_{kk}^{+} + \left[\dfrac{Z_{kk}^{-}(3Z_f + Z_{kk}^{0})}{Z_{kk}^{-} + 3Z_f + Z_{kk}^{0}}\right]}$$

$$I_a^{-} = -I_a^{+} \times \frac{3Z_f + Z_{kk}^{0}}{Z_{kk}^{-} + 3Z_f + Z_{kk}^{0}} \quad \text{------ (14)}$$

$$I_a^{0} = -I_a^{+} \times \frac{Z_{kk}^{-}}{Z_{kk}^{-} + 3Z_f + Z_{kk}^{0}}$$

Direct short Circuit or Bolted LLG Fault :-

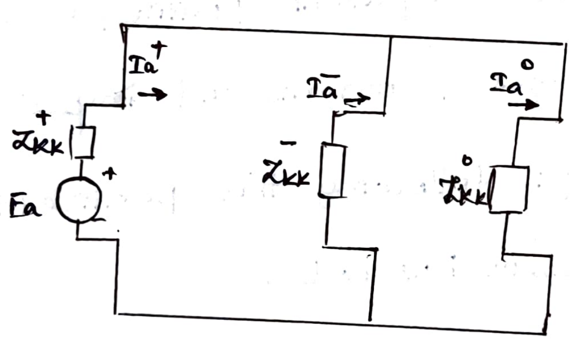


Fault impedence $Z_f = 0$.

The conditions of the fault at bus k are,

$$\left. \begin{array}{l} I_a = 0, \quad V_b = 0, \quad V_c = 0 \\ I_f = I_b + I_c \end{array} \right\} \quad \text{--------- (1)}$$

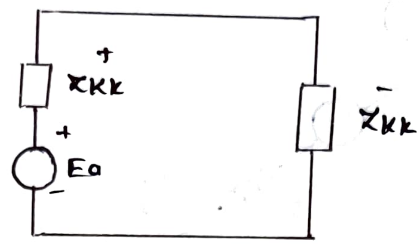
The sequence of LLG Fault,



$$\left. \begin{array}{l} I_a^+ = \dfrac{E_a}{Z_{kk}^+ + \left[\dfrac{Z_{kk}^- \times Z_{kk}^0}{Z_{kk}^- + Z_{kk}^0}\right]} \\[2ex] I_a^- = -I_a^+ \times \dfrac{Z_{kk}^0}{Z_{kk}^- + Z_{kk}^0} \\[2ex] I_a^0 = -I_a^+ \times \dfrac{Z_{kk}^-}{Z_{kk}^- + Z_{kk}^0} \end{array} \right\} \quad \text{--------- (2)}$$

Double Line to Ground Fault when $Z_f = \alpha$

when $Z_f = \alpha$, zero sequence circuit becomes an open circuit. Therefore no zero sequence current can flow.



It is similar to that of bolted line to line fault.

Short - Circuit Analysis of Unbalanced Large Scale Systems.

The method of fault analysis explained for symmetrical fault can be extended to unsymmetrical faults. The following symbols are used in unsymmetrical fault calculation.

* Superscript f represents post fault or fault values.
* Superscript +, - and o represents positive, negative and zero sequence voltages, currents and impedences.
* A number subscript following this positive (+), negative (-) and zero (0) represent bus code.
* Phase values of voltages and currents are indicated collectively by subscript p and individually by the subscript a, b and c.

· Problem statement :-

Procedure

* Assemble Thevenin's equivalent positive, negative and zero sequence networks separately using the sequence impedences of various power system components like generators, motors, transformers and transmission lines.

* Compute the positive, negative and zero sequence bus impedence matrix z^+, z^- and z^0 using bus building algorithm or short circuit fault impedence matrix Z_{bus}

* Select the type (LL, LG, LLG), location (bus number) and mathematical description of fault.

* Determine the fault current at the fault bus using the sequence network for a L-G, L-L and L-L-G faults.

* Determine the prefault sequence voltage and post fault sequence voltages.

* Compute the positive, negative and zero sequence line currents.

Network Modelling By means of Sequence Bus impedence matrices.

Consider three bus system, z^+, z^- and z^0 matrices are

$$z^+ = \begin{bmatrix} z_{11}^+ & z_{12}^+ & z_{13}^+ \\ z_{21}^+ & z_{22}^+ & z_{23}^+ \\ z_{31}^+ & z_{32}^+ & z_{33}^+ \end{bmatrix} \quad ---- \quad (1)$$

$$Z^{-} = \begin{bmatrix} Z_{11}^{-} & Z_{12}^{-} & Z_{13}^{-} \\ Z_{21}^{-} & Z_{22}^{-} & Z_{23}^{-} \\ Z_{31}^{-} & Z_{32}^{-} & Z_{33}^{-} \end{bmatrix} \quad \text{---- (2)}$$

$$Z^{0} = \begin{bmatrix} Z_{11}^{0} & Z_{12}^{0} & Z_{13}^{0} \\ Z_{21}^{0} & Z_{22}^{0} & Z_{23}^{0} \\ Z_{31}^{0} & Z_{32}^{0} & Z_{33}^{0} \end{bmatrix} \quad \text{---- (3)}$$

For both analytical and computational reasons, this network data is best assembled into an short circuit bus impedance matrix $Z_{s\,bus}$ of dimensions $3n \times 3n$, $n \rightarrow$ no of buses.

$$Z_{s.bus} = \left[\begin{array}{ccc|ccc|ccc} Z_{11}^{+} & 0 & 0 & Z_{12}^{+} & 0 & 0 & Z_{13}^{+} & 0 & 0 \\ 0 & Z_{11}^{-} & 0 & 0 & Z_{12}^{-} & 0 & 0 & Z_{13}^{-} & 0 \\ 0 & 0 & Z_{11}^{0} & 0 & 0 & Z_{12}^{0} & 0 & 0 & Z_{13}^{0} \\ \hline Z_{21}^{+} & 0 & 0 & Z_{22}^{+} & 0 & 0 & Z_{23}^{+} & 0 & 0 \\ 0 & Z_{21}^{-} & 0 & 0 & Z_{22}^{-} & 0 & 0 & Z_{23}^{-} & 0 \\ 0 & 0 & Z_{21}^{0} & 0 & 0 & Z_{22}^{0} & 0 & 0 & Z_{23}^{0} \\ \hline Z_{31}^{+} & 0 & 0 & Z_{32}^{+} & 0 & 0 & Z_{33}^{+} & 0 & 0 \\ 0 & Z_{31}^{-} & 0 & 0 & Z_{32}^{-} & 0 & 0 & Z_{33}^{-} & 0 \\ 0 & 0 & Z_{31}^{0} & 0 & 0 & Z_{32}^{0} & 0 & 0 & Z_{33}^{0} \end{array}\right] \quad \text{--- (4)}$$

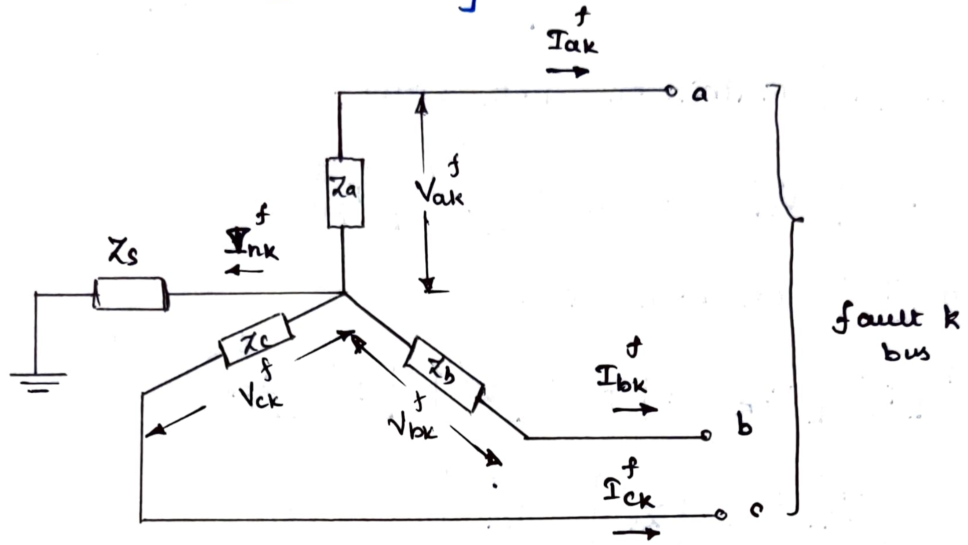
Z_s bus matrix each sub matrices of size (3×3) is diagonal with three diagonal elements equaling Z_{ij}^{+}, Z_{ij}^{-} and Z_{ij}^{0}

Fault Matrices Z_s^{f} and Y_s^{f}

In unbalanced fault analysis, we need a three dimensional vector equation for a complete fault description.

For a solid L-G fault on phase a,

$$\left.\begin{aligned} Z_a &= Z_s = \infty \\ Z_b &= Z_c = 0 \end{aligned}\right\} \quad \text{------ (1)}$$



For a solid L-L fault with simultaneous ground contact,

$$Z_a = \infty, \quad Z_b = Z_c = Z_s = 0 \quad \text{------ (2)}$$

Similarly other types of faults can be created,

$$\left.\begin{aligned} V_{ak}^f &= I_{ak}^f Z_a + I_{nk}^f Z_s = I_{ak}^f Z_a + (I_{ak}^f + I_{bk}^f + I_{ck}^f) Z_s \\ V_{bk}^f &= I_{bk}^f Z_b + I_{nk}^f Z_s = I_{bk}^f Z_b + (I_{ak}^f + I_{bk}^f + I_{ck}^f) Z_s \\ V_{ck}^f &= I_{ck}^f Z_c + I_{nk}^f Z_s = I_{ck}^f Z_c + (I_{ak}^f + I_{bk}^f + I_{ck}^f) Z_s \end{aligned}\right\} \text{- (3)}$$

In vector form,

$$\begin{bmatrix} V_{ak}^f \\ V_{bk}^f \\ V_{ck}^f \end{bmatrix} = \begin{bmatrix} Z_a + Z_s & Z_s & Z_s \\ Z_s & Z_b + Z_s & Z_s \\ Z_s & Z_s & Z_c + Z_s \end{bmatrix} \begin{bmatrix} I_{ak}^f \\ I_{bk}^f \\ I_{ck}^f \end{bmatrix} \quad \text{------ (4)}$$

In compact form.

$$V_{pk}^f = Z^f I_{pk}^f \quad \text{---------- (5)}$$

Fault impedence matrix $z^f = \begin{bmatrix} z_a + z_s & z_s & z_s \\ z_s & z_b + z_s & z_s \\ z_s & z_s & z_c + z_s \end{bmatrix}$ - - - ⑥

eqn ⑤ x [T] matrix on both sides,

$$[T][V_{pk}^f] = [z^f][T][I_{pk}^f]$$

multiply $[T^{-1}]$ on both sides,

$$[T]^{-1}[T][V_{pk}^f] = [T]^{-1}[z^f][T][I_{pk}^f]$$

$$[V_{sk}^f] = [z_s^f][I_{sk}^f] \quad - - - - - ⑦$$

↳ short circuit transformed fault matrix

$$[z_s^f] = [T]^{-1}[z^f][T]$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} z_a + z_s & z_s & z_s \\ z_s & z_b + z_s & z_s \\ z_s & z_s & z_c + z_s \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} z_a + z_b + z_c & z_a + a^2 z_b + a z_c & z_a + a z_b + a^2 z_c \\ z_a + a z_b + a^2 z_c & z_a + z_b + z_c & z_a + a^2 z_b + a z_c \\ z_a + a^2 z_b + a z_c & z_a + a z_b + a^2 z_c & z_a + z_b + z_c \end{bmatrix} \quad - - ⑧$$

$[z_s^f]$ matrix is not symmetric even through $[z_f]$ is symmetric

$$[I_{sk}^f] = [z_s^f]^{-1}[V_{sk}^f]$$

$$= [Y_s^f][V_{sk}^f]$$

$[Y_s^f]$ → fault admittance matrix

$$\therefore [Y_s^f] = [z_s^f]^{-1} \quad - - - - - - ⑨$$

Short Circuit Formulas (or) Unbalanced Fault Analysis Using Bus Impedence Matrix.

Prefault Voltages:-

Since the fault occurs when the system is balanced, all the prefault bus voltages contain only positive sequence componets.

$$V_{p.f} = \begin{bmatrix} V_1^+ \\ 0 \\ 0 \\ \vdots \\ V_i^+ \\ 0 \\ 0 \\ \vdots \\ V_n^+ \end{bmatrix} \quad \text{------ } ①$$

Post Fault Voltages:-

From Thevenin's theorem, the post fault positive sequence bus voltages are give by

$$[V_f^+] = [V_{p.f}] + [Z^+][I_f]$$

Since the fault current injected is at k-bus

$$I_f = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_k^{f+} \\ \vdots \\ 0 \end{bmatrix}$$

Post fault positive sequences bus voltages are,

$$\left.\begin{aligned} V_1^{f+} &= V_{p.f} - Z_{1k}^+ I_k^{f+} \\ &\vdots \\ V_k^{f+} &= V_{p.f} - Z_{kk}^+ \cdot I_k^{f+} \\ &\vdots \\ V_n^{f+} &= V_{p.f} - Z_{nk}^+ \cdot I_k^{f+} \end{aligned}\right\} \quad \text{------ } ②$$

Since the prefault negative, zero sequence bus voltages are zero,

the post fault negative sequence voltages and zero sequence voltages

$$\left.\begin{aligned} V_1^{f-} &= -Z_{1k}^{-} I_k^{f-} \\ &\vdots \\ V_k^{f-} &= -Z_{kk}^{-} I_k^{f-} \\ &\vdots \\ V_n^{f-} &= -Z_{nk}^{-} I_k^{f-} \end{aligned}\right\} \quad \text{--- (3)}$$

$$\left.\begin{aligned} V_1^{fo} &= V_{p\cdot f} - Z_{1k}^{o} I_k^{fo} \\ &\vdots \\ V_k^{fo} &= V_{p\cdot f} - Z_{kk}^{o} I_k^{fo} \\ &\vdots \\ V_n^{fo} &= V_{p\cdot f} - Z_{nk}^{o} I_k^{fo} \end{aligned}\right\} \quad \text{--- (4)}$$

Post fault line currents or sequence currents in the i-j line:

Positive sequence line current $I_{ij}^{f+} = \dfrac{V_i^{f+} - V_j^{f+}}{Z_{ij}^{+}}$

Negative sequence line current $I_{ij}^{f-} = \dfrac{V_i^{f-} - V_j^{f-}}{Z_{ij}^{-}}$

Zero sequence line current $I_{ij}^{fo} = \dfrac{V_i^{fo} - V_j^{fo}}{Z_{ij}^{o}}$

phase voltages :-

$$[V_p] = [T][V_s]$$

$$[T] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$[V_s] \rightarrow$ Sequence Voltage

$[V_p] \rightarrow$ phase Voltage

phase currents :-

$$[I_p] = [T][I_s]$$

$I_s \rightarrow$ sequence currents

$I_p \rightarrow$ phase currents.

Unit - V STABILITY ANALYSIS

Power system stability is the property of the system that enables it to remain in a state of operating equilibrium under normal operating conditions and to regain an acceptable state of equilibrium after being subjected to a disturbance.

Steady state	Transient state
1) All the measured physical quantities describing the operating conditions are constant for the analysis.	1) The measured quantities are not constant.
2) when it follows small disturbance, and it returns to the same steady state conditions.	2) when it follows large disturbance and a significantly different but acceptable steady state operating condition is attained.
3. It can be analysed by linear equation. The non-linear equations are replaced by linear equations.	3) It can be analysed by using non linear equations.
4) Ex. Change in gain of AVR in excitation system of a large generating unit.	4) Ex. Transmission system fault, sudden load changes, line switching, loss of gen. unit

Power System Stability Problem :-

Stability problem is concerned with the behaviour of the synchronous machine after a disturbance.

Stability problem may be divided into steady state stability and transient stability.

Steady state stability :-

It is the stability of the power system to bring it to a stable condition or remains in synchronism after a small disturbance such as a gradual infinitesimal variation in system variable like rotor angle, voltage, etc. and its classification,

(i) Static stability

It refers to inherent stability that prevails without the aid of automatic control devices.

(ii) Dynamic stability.

It refers to inherently unstable system with automatic control devices.

Transient stability :-

It is the ability of the system to bring it to a stable condition after a large disturbance. Large disturbance can occure due to the occurance of fault, sudden outage of line, sudden loss of excitation, sudden application or removal of loads, etc

Importance of stability analysis in power system planning and operation :-

* Transient stability studies deal with the effects of large, sudden disturbances such as the occurance of a fault, sudden outage of a line or the sudden application or removal of loads.
* Transient stability studies gives the information that the system can withstand the transient conditions like high magnitude of voltage and frequency.
* It deals with the stability of the system.
* Transient stability studies are needed when the new generating station and transmission facilities are planned.
* It is useful in determining the nature of the relaying system needed, critical clearing time of circuit breakers, i.e design of protection equipments.
* It is more helpful in determining power system of transfer capability between two different systems.

Causes, Nature and Effect of disturbances :-

* Natural causes such as a tornado that can cause a flashover across insulators
* In advertent causes such as maloperation of protection
* Intended actions such as opening/closing of circuit breakers by the operator.

Classification of power System stability:-

The power system stability was classified as angle stability, voltage stability and frequency stability. Classification of stability based on the following considerations

* The physical nature of the resulting instability.
* The size of the disturbance considered.
* The devices ~~and~~, processes and time span that must be taken into consideration in order to determine stability.
* Prediction of stability.

1) **Rotor angle stability:-**

It is the ability of interconnected synchronous machines of a power system to remain in synchronism.

The stability problem involves the study of the electro-mechanical oscillations involving exchange of energy between network and generator - mechanical system at or close to power frequency. The problem is the manner in which the output power of the synchronous machines vary as rotor oscillates.

The rotor angle stability phenomena can be divided into two categories.

→ small signal (or small signal stability)

→ Transient stability or large signal (large disturbance) stability.

(ii) Small signal stability:-

It is the ability of the power system to maintain synchronism under small disturbances. Such disturbances occure continuously on the system because of small variations in the loads and generation. In ability may result due to the following two forms.

(i) steady increase in rotor angle due to lack of sufficient synchronising torque.

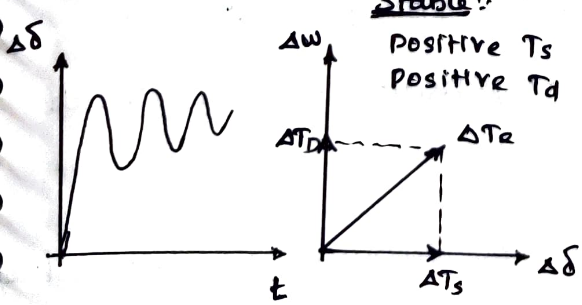
(ii) Rotor oscillations of increasing amplitude due to lack of sufficient damping torque.

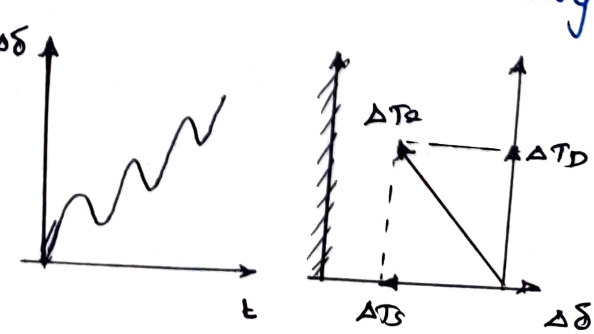
The nature of the system response to small disturbance depends on a number of factors including the initial operating, the transmission system strength, and the type of generator excitation controls used.

For generator connected radially to a large power system, the absense of automatic voltage regulators (AVR) the stability is due to lack of sufficient synchronising torque. This results in instability through a non-oscillatory mode.

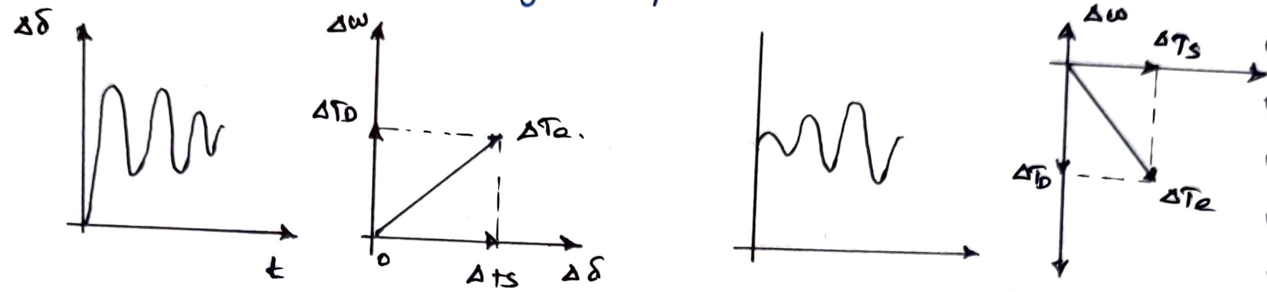
Stable:-

Positive T_s
Positive T_d





With continuously acting voltage regulators, the small disturbance stability problem is one of ensuring sufficient damping of oscillations. Instability is normally through oscillations of increasing amplitude.



Stability Types of oscillations :-

* Local mode or machine system modes are associated with swinging of units at a generating station with respect to the rest of the power system. The term local is used because the oscillations are localized at one station or small part of the power system.

* Inter area modes are associated with swinging of many machines in one part of system against machines in other parts. They are caused by two or more groups of closly coupled machine being interconnected by weak ties.

* control modes are associated with generating units and other controls. Poorly tuned excitors, speed governors, HVDC convertors and static VAR compensators are the usual causes of instability of these modes.

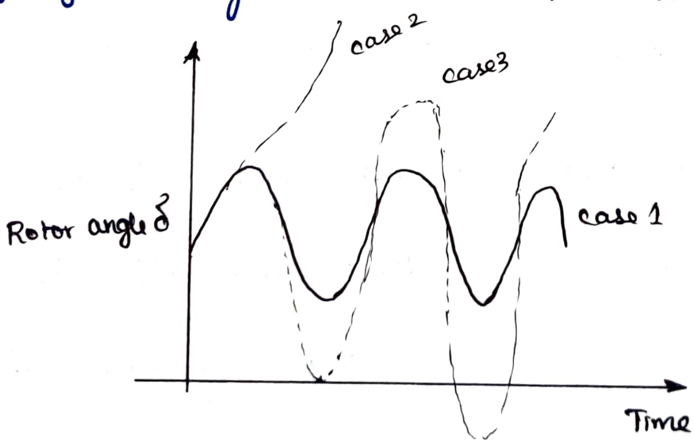
* Torsional modes are associated with the turbine generator shaft

system rotational components. Instability of torsional mode may be caused by interaction with excitation controls, speed governors, HVDC controls and series capacitor-compensated lines.

Transient Stability or Large Signal Stability :-

* It is the ability of power system to maintain synchronism when subjected to severe transient disturbance.
* The resulting system response involves large excursions of generator rotor angles and is influenced by the non-linear power angle relationship.
* Stability depends on both initial operating state of the system and the severity of the disturbance.

Ex : Transmission system faults, sudden load changes, loss of generating units and line switching.



case (i) : In the stable case, the rotor angle increases to a maximum, then decreases and oscillates with decreasing amplitude until it reaches a steady state.

case (ii) : The rotor angles continue to increases steadily untill synchronism is lost. This form of instability is referred to as first swing instability and is caused by insufficient synchronizing torque.

case (iii) : The system is stable in the first swing but becomes unstable as a result of growing oscillations at the end state is approached. This form of instability generally occurs when the post fault steady state condition itself is "small signal" unstable and not necessarily as a result of the transient disturbance.

In large power systems, transient instability may not always occurs as first swing instability. It could be the result of the superposition of several modes of oscillation causing large excursions of rotor angle beyond the first swing.

In this study, the period of interest is usually limitted to 3 to 5 sec. following the disturbance, although it may extend to about 10 sec. for very large system with dominent inter area modes of oscillations.

Voltage stability:-

It is the ability of a power system to maintain steady acceptable voltages at all buses in the system under normal operating conditions and after being subjected to a disturbance.

Causes:-

* Increase in load demand
* change in system condition
* progressive and uncontrollable voltage drop.
* Inability of power system to meet demand for reactive power.
* Bus voltage magnitude increases as the reactive power injection at the same bus is increased.

Classifications:-

(i) Large disturbance voltage stability:-

It is concerned with a system stability to control voltages following large disturbances such as system faults, loss of generation, or circuit contingencies. This ability is determined by system load characteristics and the interaction of both continuous and discrete controls and protections.

(ii) **Small Disturbance voltage Stability:-**

It is concerned with the system stability to control voltages following small perturbations such as incremental changes in system load continuous controls and discrete controls at a given instant of time.

SINGLE MACHINE INFINITE BUS (SMIB) SYSTEM

Rotor Dynamics and Swing Equation:-

Power or Torque angle:-

Under normal operating conditions, the relative position of the rotor axis and the result magnetic field are fixed. The angle between the two is known as the power angle or torque angle δ.

Swing Equation:-

During any disturbance, rotor will decelerate with respect to the synchronously rotating airgap mmf, and a relative motion begins. The equation used to describe the behaviour of the synchronous machine during transient period is known as the swing equation.

After oscillatory period, the rotor locks back into synchronous speed, the generator will maintain its stability. If the disturbance is created by change in generation, load in network conditions, the rotor comes to a new operating power angle relative to the synchronously revolving field.

Assumptions in Stability Studies:

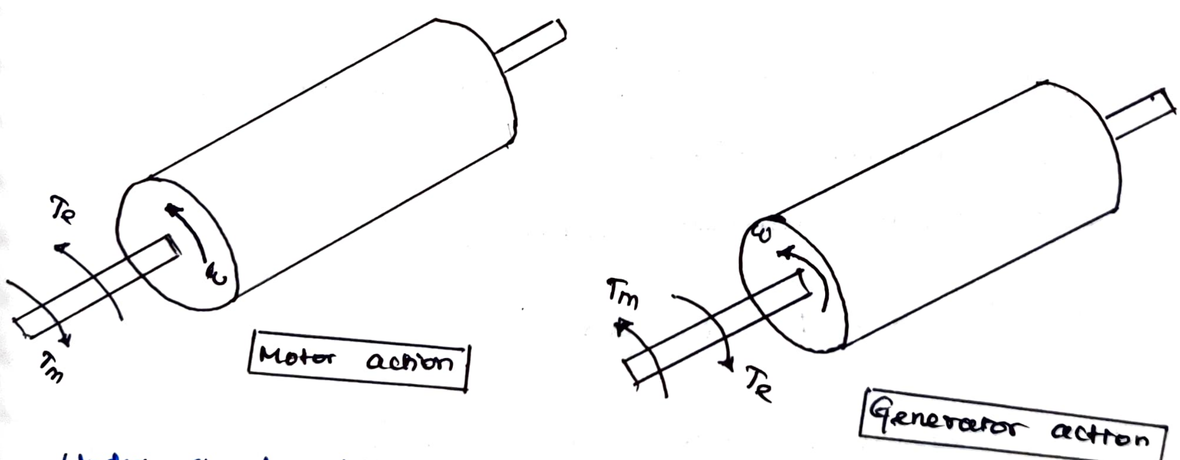
* Machine represented by classical model.
* Controllers are not considered.
* Loads are constant.
* Voltage and currents are sinusoidal.

Consider a synchronous generator developing an electromagnetic Torque T_e and running at the synchronous speed ω_{sm}.

Let $T_m \rightarrow$ driving mechanical torque.

$T_e \rightarrow$ electrical torque.

For generator action, T_m and T_e are positive. θ_m positive.



Under steady state with losses neglected.

$$T_m = T_e$$

Accelerating torque $T_a = T_m - T_e = 0$

No acceleration or deceleration of rotor. Due to disturbance results in an accelerating ($T_m > T_e$) or decelerating ($T_m < T_e$) torque on the rotor.

Accelerating torque $T_a = T_m - T_e$.

Let J be the moment of inertia of the primemover and generator.

From laws of rotation,

Acceleration $\alpha = \dfrac{d^2\theta_m}{dt^2}$

Accelerating torque $T_a = J \times \alpha$

$$J \frac{d^2\theta_m}{dt^2} = T_m - T_e \quad \text{--- --- --- } (1)$$

where θ_m is the angular displacement of the rotor with respect to the stationary reference axis on stator.

θ_m increases with time even at constant synchronous speed

$$\theta_m = \omega_{sm} t + \delta_m \quad \text{--- --- --- --- } (2)$$

$\delta_m \rightarrow$ Angular displacement of the rotor before disturbance in mechanical radians.

$\omega_{sm} \rightarrow$ constant angular velocity.

diff. eqn (2) w.r.t t, we get,

Rotor angular velocity $\omega_m = \dfrac{d\theta_m}{dt} = \omega_{sm} + \dfrac{d\delta_m}{dt}$ --- --- --- (3)

diff. eqn (3) w.r.t to t, rotor acceleration is

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

Substituting in eqn ①, we get

$$J.\frac{d^2\delta_m}{dt^2} = T_m - T_e$$

multiplying by ω_m on both sides,

$$J.\omega_m \frac{d^2\delta_m}{dt^2} = \omega_m T_m - \omega_m T_e.$$

Inertia constant :-

M - constant or inertia constant is defined as the angular momentum at synchronous speed. If energy is measured in Joules and speed in mechanical radians per second. Unit of M is Joule - sec/Mechanical radian.

$M = J \times \omega_m$ is the inertia constant.

Angular Momentum of the rotor at sychronous speed.

$$M.\frac{d^2\delta_m}{dt^2} = P_m - P_e \quad (P = \omega T) \text{ --- } ④$$

where P_m, P_e are mechanical and electrical power.

This is the swing equation in terms of inertia constant.

P.U Inertia Constant :-

Kinetic Energy of the rotating Masses $W_k = \frac{1}{2} J \omega_m^2$

For stability studies, 'Per unit inertia constant H' defined as,

$$\text{P.U Inertia constant} = \frac{\text{Stored kinetic energy in Mega Joules of turbine alternator and exciter rotor at syn. speed.}}{\text{Machine rating in MVA.}}$$

$$H = \frac{\frac{1}{2} J \omega_{sm}^2}{S_B} \text{ sec.}$$

$$J \omega_{sm} = \frac{2 H S_B}{\omega_{sm}} = M.$$

Substituting in eqn ④,

$$\frac{2 H S_B}{\omega_{sm}} \cdot \frac{d^2 \delta_m}{dt^2} = P_m - P_e. \quad \text{---} ⑤$$

With angle and speed on electrical side,

$\omega_{se} = 2\pi f$, $\omega_{sm} = \frac{2\pi N_s}{6}$

$\omega_{sm} = \frac{2\pi \times 120f}{60 P}$

$\omega_{sm} = \frac{2 \omega_{se}}{P}$

$\int \omega_{sm} = \delta_m = \frac{2\delta}{P}$

$$\frac{2 H S_B}{\frac{2}{P} \times \omega_{se}} \times \frac{d^2 \delta_m}{dt^2} = P_m - P_e$$

$$\frac{2 H S_B}{\frac{2}{P} \times 2\pi f} \times \left(\frac{2}{P}\right) \frac{d^2 \delta}{dt^2} = P_m - P_e$$

$$\frac{H S_B}{\pi f} \times \frac{d^2 \delta}{dt^2} = P_m - P_e \quad \text{---} ⑥$$

Dividing by MVA rating S_B on both sides of eqn ⑥,

$$\frac{H}{\pi f} \times \frac{d^2 \delta}{dt^2} = \frac{P_m}{S_B} - \frac{P_e}{S_B}$$

$\frac{P_m}{S_B} \rightarrow$ Per unit mechanical power.

$\frac{P_e}{S_B} \rightarrow$ Per unit electrical power.

$$\frac{H}{\pi f} \times \frac{d^2 \delta}{dt^2} = P_{m(p.u)} - P_{e(p.u)} = P_{m(p.u)} - P_m . \sin\delta \quad \text{---} ⑦$$

$$M_{(p.u)} \times \frac{d^2 \delta}{dt^2} = P_{m(p.u)} - P_{e(p.u)}$$

where $M_{(p.u)} = \frac{H}{\pi f}$. δ in radians.

If δ is expressed in electrical degrees.

$$\frac{H}{180 f} \times \frac{d^2\delta}{dt^2} = P_{m(p.u)} - P_{e(p.u)}$$

These equations are called as swing equations.

Swing Curve :-

From eqn (5), as two first order equations.

$$\frac{2H}{\omega_{sm}} \times \frac{d\Delta\omega}{dt} = P_{m(p.u)} - P_{e(p.u)}$$

$$\Rightarrow \frac{d\Delta\omega}{dt} = \frac{\pi f_0}{H}\left[P_m - P_{max}\sin\delta\right]$$

$$\frac{d\delta}{dt} = \omega_{sm} - \omega_{se} = \Delta\omega$$

$\Delta\omega \rightarrow$ rotor speed deviation in p.u.

The graphical display of δ versus t is called the swing curve.

The plot of swing curves of all machines tells us whether machines will remain in synchronism after a disturbance.

Typical Value of H :-

Type of Machine		H in MJ/MVA
Turbo Generator		
Condensing	1800 rpm	9 - 6
	3600 rpm	7 - 4
Non-condensing	3600 rpm	4 - 3
Water wheel Generator :-		
Slow speed	< 200 rpm	2 - 3
high speed	> 200 rpm	2 - 4

POWER ANGLE EQUATION

This equation relating the electrical power generated (P_e) to the angular displacement of the rotor (δ) is called power angle equation.

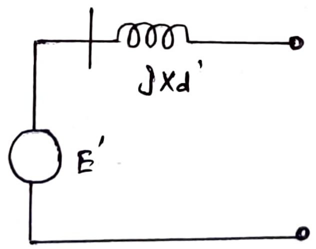
Assumptions:

* Mechanical power input P_m is constant during the period of electromechanical transient, ie effect of governer action is neglected.

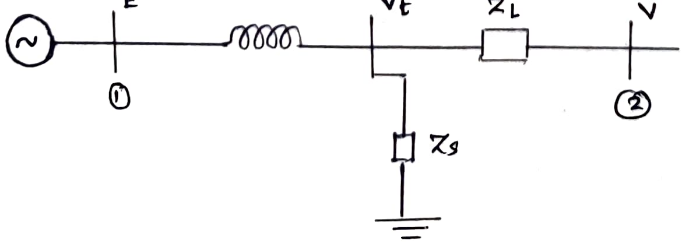
* Rotor speed changes are insignificant.

* The generated machine emf remains constant -ie. Effect of voltage regulating loop is neglected.

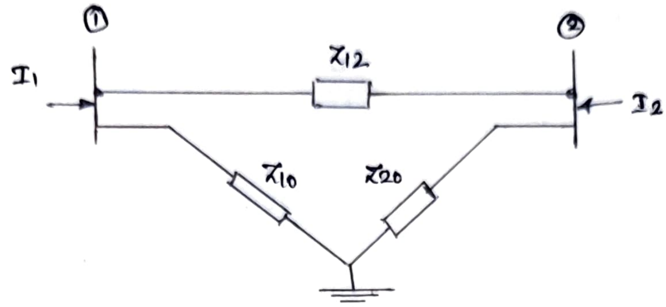
Synchronous machine model neglecting saliency is the simplest classical model for stability analysis. Here the machine is represented by a constant voltage E' behind the direct axis transient reactance X_d'.



Consider a generator connected to a major substation of a very large system (infinite bus) through a transmission line.



Eliminate the generator terminal voltage (V_t) node by using Y - Δ transformation.



$$Z_{12} = \frac{jX_d' Z_L + jX_d' Z_L + Z_L Z_S}{Z_S} \quad , \quad Z_{10} = \frac{jX_d' Z_L + jX_d' Z_S + Z_L Z_S}{Z_L}$$

$$Z_{20} = \frac{jX_d' Z_L + jX_d' Z_S + Z_L Z_S}{jX_d}$$

Nodal equation,

Node 1, $I_1 = \left[\frac{1}{Z_{12}} + \frac{1}{Z_{10}}\right] E_s' - \frac{1}{Z_{12}} V$

Node 2, $I_2 = -\frac{1}{Z_{12}} E' + \left[\frac{1}{Z_{12}} + \frac{1}{Z_{20}}\right] V$

$$I_1 = Y_{11} E_o' + Y_{12} V$$

$$I_2 = Y_{21} E' + Y_{22} V$$

power injected at bus 1,

$$P_1 + jQ_1 = E' I_1^*$$

$$= E' [Y_{11} E']^* + E' [Y_{12} V]$$

$$= E' \angle\delta \, [Y_{11} \angle -\theta_{11} \cdot E' \angle -\delta + E' \angle\delta \times Y_{12} \angle -\theta_{12} \, V \angle 0$$

$$= E'^2 Y_{11} \angle -\theta_{11} + E' V Y_{12} \angle \delta - \theta_{12}$$

$$P_1 = \text{Real} \{ P_1 + jQ_1 \}$$

$$= E'^2 Y_{11} \cos\theta_{11} + E' V Y_{12} \cos(\delta - \theta_{12})$$

$$P_1 = E'^2 G_{11} + E' V Y_{12} \cos(\delta - \theta_{12}).$$

Mostly Z_L and X_s are inductive, so resistance are neglected..

$$\theta_{12} = 90^\circ, \qquad Y_{12} = \frac{1}{X_{12}}$$

$$P_1 = P_e = E'^2 G_{11} + \frac{E'V}{X_{12}} \sin\delta$$

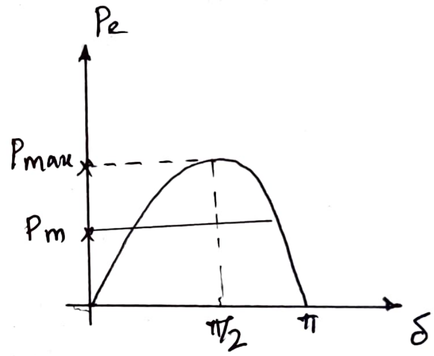
$$P_e = P_c + P_{max} \sin\delta$$

This is called power angle equation.

Power angle Curve :-

All the succeptance are having elements with $G_{11} = 0$.

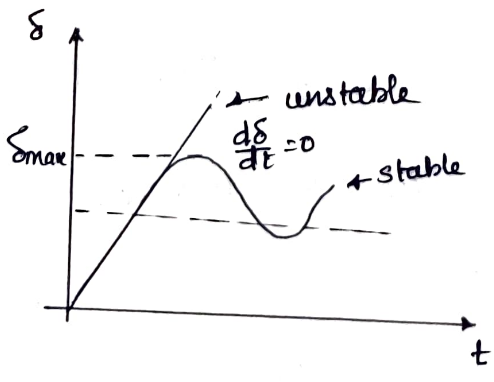
$$P_e = \frac{E' \times V}{X_{12}} \sin\delta = P_{max} \sin\delta$$



Power transmitted depends on the transfer reactance X_{12} and angle between the voltages E' and V. ie(δ). The curve P_e versus δ is known as power angle curve.

EQUAL AREA CRITERION

The equal area criterion for stability states that the System is stable if the area under $P_a - \delta$ curve reduces to zero at some value of δ.



This is possible if the possitive (accelerating) area under $P_a - \delta$ curve is equal to the negative (decelerating) area under $P_a - \delta$ curve for a finite change in δ. Hence the stability criterian called equal area criterion.

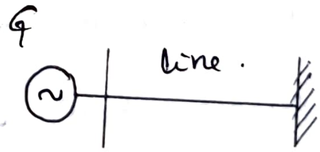
This method is only applicable to one machine connected to infinite bus or two machine system.

Stability criterian :-

System stable :- If the system is stable, $\delta(t)$ perform oscillations whose amplitude decreases in actual practice. At some time, $\frac{d\delta}{dt} = 0$, δ reaches maximum and will start to reduce.

unstable system :- If the system is unstable, δ continues to increase with time and the machine loss synchronism. $\frac{d\delta}{dt} > 0$ for a sufficiently long time.

The swing equation is given by,



$$\frac{H}{\pi f} \times \frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}\left[P_m - P_e\right]$$

Multiplying equation by $2\frac{d\delta}{dt}$ on both sides, we get.

$$2 \cdot \frac{d\delta}{dt} \times \frac{d^2\delta}{dt^2} = \frac{2\pi f}{H}(P_m - P_e) \cdot \frac{d\delta}{dt}$$

This may be written as,

$$\frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right] = \frac{2\pi f}{H}(P_m - P_e) \times \frac{d\delta}{dt}$$

$$d\left[\frac{d\delta}{dt}\right]^2 = \frac{2\pi f}{H}(P_m - P_e) \times d\delta$$

integrating both sides, we get

$$\left[\frac{d\delta}{dt}\right]^2 = \frac{2\pi f}{H}\int_{\delta_0}^{\delta}(P_m - P_e)\, d\delta .$$

$$\left.\begin{array}{l}\text{Relative speed of the machine} \\ \text{with respect to synchronously} \\ \text{revolving reference frame}\end{array}\right\} \frac{d\delta}{dt} = \left[\frac{2\pi f}{H}\int_{\delta_0}^{\delta}(P_m - P_e)\, d\delta\right]^{1/2}$$

For stable system, this speed must become zero at some time after the disturbance.

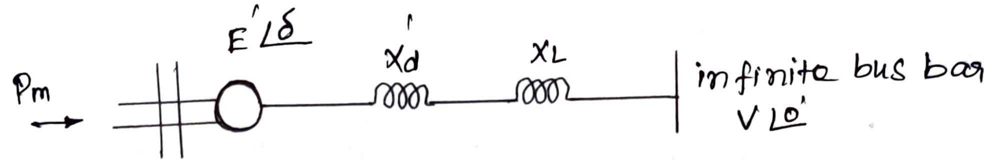
$$\frac{d\delta}{dt} = 0, \quad \int_{\delta_0}^{\delta}(P_m - P_e)\, d\delta = 0$$

$$\int_{\delta_0}^{\delta} P_a\, d\delta = 0 \quad , \quad P_a \rightarrow \text{accelerating power.}$$

The condition of stability can be stated as the positive (accelerating) area under P_a vs δ curve must equal to the negative (decelerating) area and hence the name equal area criterion of stability.

Sudden change in Mechanical Input :-

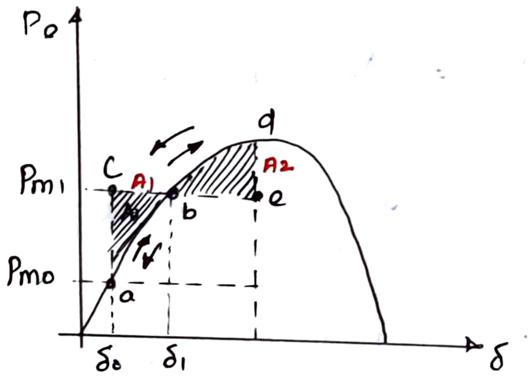
Consider the transient model of single generator connected to infinite bus and a sudden step increase in input mechanical power.



Electrical power transmitted $P_e = \frac{E' \times V}{X_d' + X_e} \sin\delta = P_{max} \sin\delta$

Under steady state condition,

$$P_{m0} = P_{e0} = P_{max} \sin \delta_0$$



A sudden step increase in input power represented by the horizondal line, P_{m1}. Since $P_{m1} > P_e$, the accelerating power P_e, $P_a = P_{m1} - P_e$ on the rotor is positive and the power angle δ increases and the rotor speed increases. The excess energy stored in the rotor during the initial acceleration is,

$$\int_{\delta_0}^{\delta_1} (P_{m1} - P_e)\, d\delta = \text{Area } abc = \text{Area } A_1$$

when $\delta = \delta_1$, At point b, the electrical power matches the new input power P_{m1}.

$P_a = P_{m1} - P_e = 0$, the rotor is running above synchronous speed. Hence δ and P_e will continue to increase.

When $P_{m1} < P_e$, P_a is ~~positive~~ negative, the rotor decelerates towards synchronous speed but the angle increases upto δ_{max} indicated at point d. The energy given up by the rotor during deceleration is

$$\int_{\delta_1}^{\delta_{max}} (P_{m1} - P_e)\, d\delta = \text{Area bde} = \text{Area } A_2 .$$

At point b, the decelerating area A_2 equals the decelerating area A_1.

$$\text{Area } A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e)\, d\delta = 0 .$$

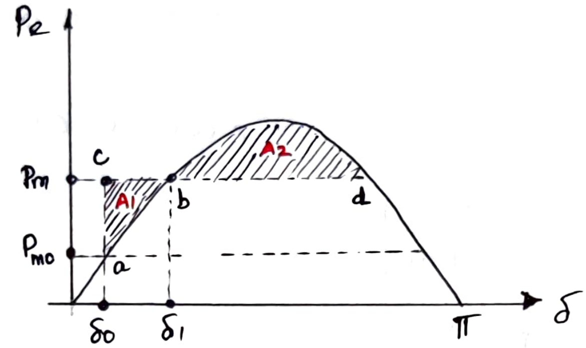
$$\int_{\delta_0}^{\delta_1} (P_{m1} - P_e)\, d\delta + \int_{\delta_1}^{\delta_{max}} (P_{m1} - P_e)\, d\delta = 0$$

$$\text{Area } A_1 = \text{Area } A_2 .$$

This is equal area criterion. The speed reduces below N_s and the δ reduces. The rotor angle would then oscillate back and forth between δ_0 and δ_{max} at its natural frequency. Damping present in the machine will cause these oscillation to subside and a new steady state operation would be established at point b.

Application to sudden increase in power input :-

The equal area criterion is used to determine the maximum additional power P_m which can be applied for stability to be maintained and δ_{max}. The stability is maintained only when area A_2 atleast equal to area A_1 can be located above P_m.



If area $A_2 <$ Area A_1, the accelerating momentum can never be over come. The limit of stability occurs when δ_{max} is at the intersection of P_m and the power angle curve for $90° < \delta < 180°$.

Applying equal area criterion, we have area $A_1 =$ Area A_2

$$P_m(\delta_1 - \delta_0) - \int_{\delta_0}^{\delta_1} P_{max} \sin\delta \, d\delta = \int_{\delta_1}^{\delta_{max}} P_{max} \sin\delta \, d\delta - P_m(\delta_{max} - \delta_1)$$

$$P_m(\delta_1 - \delta_0 + \delta_{max} - \delta_1) = \int_{\delta_1}^{\delta_{max}} P_{max} \sin\delta \, d\delta + \int_{\delta_0}^{\delta_1} P_{max} \sin\delta \, d\delta.$$

$$P_m(\delta_{max} - \delta_0) = P_{max}\left[-\cos\delta\right]_{\delta_1}^{\delta_{max}} + P_{max}\left[-\cos\delta\right]_{\delta_0}^{\delta_1}$$

$$P_m\left[\delta_{max} - \delta_0\right] = P_{max}\left[-\cos\delta_{max} + \cos\delta_1 - \cos\delta_1 + \cos\delta_0\right]$$

$$P_m\left[\delta_{max} - \delta_0\right] = P_{max}\left[\cos\delta_0 - \cos\delta_{max}\right]$$

At point d, $P_m = P_{max} \sin\delta_{max}$

Substitude P_m value in above equation,

$$P_{max} \sin \delta_{max} [\delta_{max} - \delta_0] = P_{max} [\cos \delta_0 - \cos \delta_{max}]$$

$$\sin \delta_{max} [\delta_{max} - \delta_0] + \cos \delta_{max} - \cos \delta_0 = 0$$

The non linear equation can be solved and δ_{max} can be obtain. Maximum permissible power or the transient stability limit can be found. From the graph, at point b,

$$P_m = P_{max} \sin \delta_1 \quad , \text{where } \delta_1 = \pi - \delta_{max}.$$

$$\delta_1 = \sin^{-1} \left[\frac{P_M}{P_{max}} \right]$$

Further increase in P_m, the area available for A_2 is less than area A_1, so the excess kinetic energy causes δ to increase beyond the point d. The decelerating power changes over to accelerating power so the system becomes unstable. By the use of equal area criterion, there is an upper limit to sudden increase in mechanical input ($P_m - P_{m0}$).

Thus the system is remains stable. From the graph, the system stable even though the rotor may oscillate beyond $\delta = 90°$ until equal area criterion is met.

The condition $\delta = 90°$ is meant for use in steady state stability and doesnot apply for transient stability

Application to 3φ Fault at the sending end:-

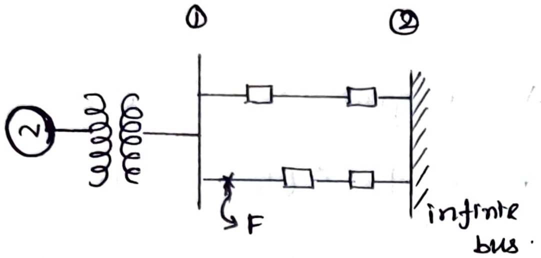
If a three phase fault occurs at point F of the outgoing radial line at bus 1. The electrical output reduces to zero ($P_e = 0$) and the point drops to b in the curve.

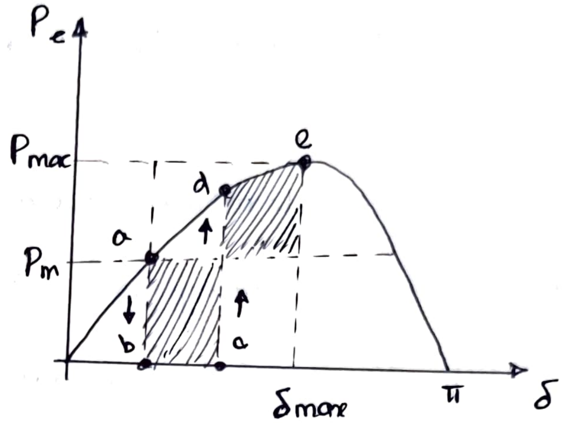
The acceleration area A_1 begins to increase and the point moves along bc. At time t_c (Clearing time) corresponding to angle δ_c (clearing angle), the faulted line is cleared by the opening of the circuit breaker.

The values of t_c and δ_c are known as clearing time and clearing angle. The system is once again becomes healthy and transmits $P_e = P_{max} \sin\delta$ ($P_e > P_m$). ie the point c shifts to d on the origional power angle curve.

The rotor now decelerates and decelerating area A_2 begins, while the point moves along de and the path is retraced along the curve passing P_e through points d and a.

For any given initial load in the case of a fault clearences on a synchronous machine connected to an infinite bus bar, there is a critical clearing angle. If the actual clearing angle is greater than the critical value, the system is unstable, otherwise the system is stable. Maximum allowable time for a system to remain stable are known as critical clearing time.





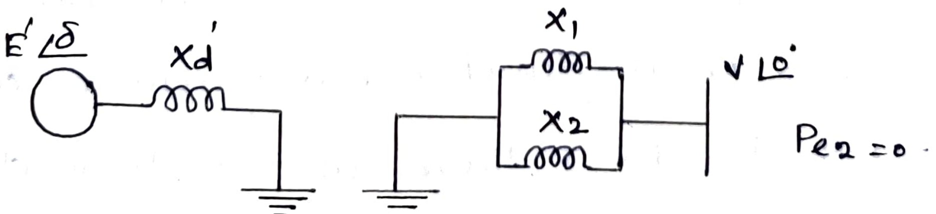
If an angle δ_1 can be found that Area A_1 = Area A_2, the system is found to be stable. The system finally settles down to the steady operating point a in an oscillatory manner because of inherent damping. At point a, $P_m = P_e$.

Prefault condition :-

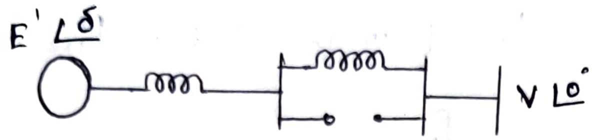
Power angle equation $P_{e_1} = \dfrac{E' \times V}{X_d' + \left[\dfrac{X_1 \times X_2}{X_1 + X_2}\right]} \times \sin\delta = P_{max} \sin\delta$

During Fault condition :-

The generator gets isolated from the power system for purposes of power flow.



Post Fault Condition :- The circuit breaker at the two ends of the faulted line open at time t_{cr} disconnecting the faulted line.



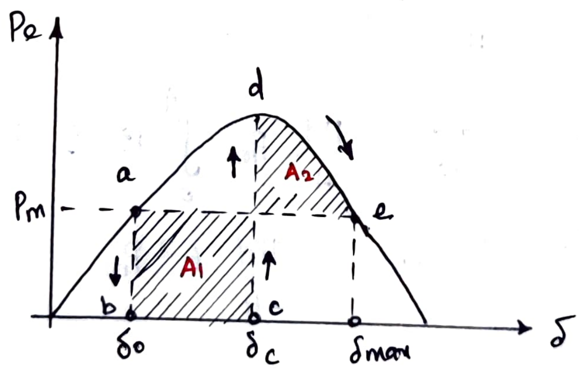
power angle equation is given by

$$P_{e3} = \frac{E' \times V}{X_d' + X_1} \sin\delta = P_{max3} \sin\delta$$

Determination of critical clearing angle and clearing time :-

Now , $P_{max2} < P_{max1}$

The critical clearing angle is reached when any further increase in δ_c causes the area A_2 < area A_1 . This occurs when δ_{max} or point c is at the intersection of line P_m and curve P_e.



Apply equal area criterion , Area A_1 = Area A_2

$$\int_{\delta_0}^{\delta_c} P_m \, d\delta = \int_{\delta_c}^{\delta_{max}} (P_{max} \sin\delta - P_m) \, d\delta$$

$$P_m \left[\delta\right]_{\delta_0}^{\delta_c} = P_{max} \left[(-\cos\delta) - P_m \delta\right]_{\delta_c}^{\delta_{max}}$$

$$P_m \delta_c - P_m \delta_0 = - P_{max} \cos \delta_{max} + P_{max} \cos \delta_c - P_m \delta_{max} + P_m \delta_c$$

Solving for δ_c, we get

$$P_{max} \cos \delta_c = P_m (\delta_{max} - \delta_0) + P_{max} \cos \delta_{max}$$

Dividing by P_{max}, $\cos \delta_c = \frac{P_m}{P_{max}} (\delta_{max} - \delta_0) + \cos \delta_{max}$.

The maximum allowable value of clearing time and angle for a system to remain stable are known as critical clearing time (t_{cr}) and critical clearing angle (δ_{cr}).

For stable system, $\cos \delta_{cr} = \frac{P_m}{P_{max}} (\delta_{max} - \delta_0) + \cos \delta_{max}$

During 3ϕ fault, $P_e = 0$, the swing equation becomes,

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_m.$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H} P_m.$$

integrating on both sides, $\frac{d\delta}{dt} = \frac{\pi f}{H} P_m \int_0^t dt = \frac{\pi f P_m t}{H}$

At, $\delta = \frac{\pi f P_m}{H} \int_\delta^t t\, dt = \frac{\pi f}{2H} t^2 P_m + \delta_0$

$\delta = \delta_{cr}$, $t = t_{cr}$

$$\therefore \delta_{cr} = \frac{\pi f t_{cr}^2 P_m}{2H} + \delta_0$$

$$t_{cr} = \sqrt{\frac{2H}{\pi f P_m} (\delta_{cr} - \delta_0)}$$

$H \rightarrow$ p.u inertia constant, $f \rightarrow$ frequency, $P_m \rightarrow$ Mechanical Power

$\delta_{cr} \rightarrow$ critical clearing angle, $\delta_0 \rightarrow$ rotor angle.

Determination of critical clearing time by Trial and Error Method.

Critical clearing time is the maximum allowable time between the occurance of a fault and clearing of the fault for which the system will be stable.

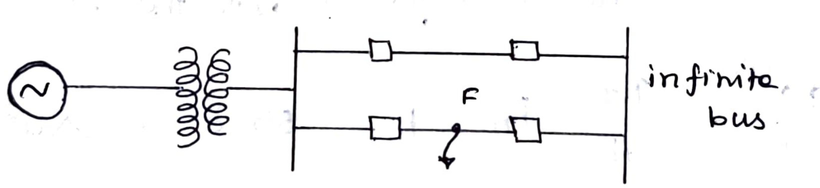
For a given load condition and specified fault, critical clearing time for a system is found out by trial and error method.

Critical time margin = critical clearing time − clearing time specified.

Modified Euler Method:-

Algorithm for Numerical solution of swing Equation using modified Euler Method:-

Numerical integration techniques can be applied to obtain approximate solution of non-linear differential equations.



$P_m \rightarrow$ input power (constant)

Prefault condition:- Under steady state operation, Power transfer from generator to an infinite bus.

$$P_e = P_m$$

$$\frac{E' \times V}{X_1} \sin \delta_0 = P_{max1} \sin \delta_0 = P_m$$

$$\sin \delta_0 = \frac{P_m}{P_{max1}} \Rightarrow \delta_0 = \sin^{-1}\left[\frac{P_m}{P_{max1}}\right]$$

$$P_{max1} = \frac{E' \times V}{X_1} \quad , \quad X_1 \rightarrow \text{Transfer reactance for the prefault condition}$$

The rotor running at synchronous speed.

$$\omega_0 = 2\pi f$$

change in angular velocity is zero , $\Delta\omega_0 = 0$.

<u>During the fault :-</u>

Consider a 3ϕ fault occurs at the middle of one line.

$$P_{e2} = \frac{E' \times V}{X_{11}} \sin\delta_1 = P_{max2} \sin\delta$$

where $P_{max2} = \dfrac{E' \times V}{X_{11}}$

$X_{11} \rightarrow$ transfer reactance during the fault.

The swing equation is given by

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}\left[P_m - P_{max2} \sin\delta\right] = \frac{\pi f}{H} P_a$$

The above equation are transformed into the state variable form

$$\frac{d\delta^{(1)}}{dt} = \frac{\pi f}{H} P_m - P_{max2} \sin\delta = \Delta\omega$$

$$\frac{d^2\delta}{dt^2} = \frac{d\Delta\omega^{(1)}}{dt} = \frac{\pi f P_a}{H}$$

Compute the first estimate at $t_1 = t_0 + \Delta t$

$$\delta^{P}_{t+1} = \delta_1 + \left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_1} \cdot \Delta t$$

$$\Delta\omega^{P}_{t+1} = \Delta\omega_1 + \left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_1} \cdot \Delta t$$

Compute the derivatives:- Using the predicted values δ_{i+1}^{P} and $\Delta\omega_{i+1}^{P}$, determine the derivatives at the end of iteration.

$$\left.\frac{d\delta^{(2)}}{dt}\right|_{\Delta\omega_{i+1}^{P}} = \Delta\omega_{i+1}^{P}$$

$$\left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta_{i+1}^{P}} = \frac{\pi f}{H} P_a \Big|_{\delta_{i+1}^{P}}$$

Compute the average derivatives,

$$\frac{d\delta}{dt_{ave}} = \frac{\left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta^{(2)}}{dt}\right|_{\Delta\omega_{i+1}^{P}}}{2}$$

$$\frac{d\Delta\omega}{dt_{ave}} = \frac{\left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta_{i+1}^{P}}}{2}$$

Compute the final estimate

$$\delta_{i+1}^{C} = \delta_i + \left[\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{i+1}^{P}}}{2}\right]\Delta t$$

$$\Delta\omega_{i+1}^{C} = \Delta\omega_i + \left[\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{i+1}^{P}}}{2}\right]\Delta t.$$

Ranga - Kutta Method:-

Steps are involved in Range kutta method to determine stability,

I estimates: $k_1 = \left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} \times \Delta t = \Delta\omega_i \times \Delta t$

$l_1 = \left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} \times \Delta t = \frac{\pi f}{H}\left[P_m' - P_e(\delta_i)\right]\Delta t$

II Estimate :
$$k_2 = \left[\Delta\omega_i + \frac{l_1}{2}\right]\Delta t$$

$$l_2 = \frac{\pi f}{H}\left[P_m' - P_e\left(\delta_1 + (k_1/2)\right)\right] \times \Delta t$$

III Estimate :
$$k_3 = \left(\Delta\omega_i + \frac{l_2}{2}\right)\Delta t$$

$$l_3 = \frac{\pi f}{H}\, P_m' - P_e\left(\delta_i + (k_2/2)\right)\Big] \times \Delta t$$

IV Estimate :
$$k_4 = (\Delta\omega_i + l_3) \times \Delta t$$

$$l_4 = \frac{\pi f}{H}\left[P_m' - P_e(\delta_i + k_3)\right] \times \Delta t$$

Final Estimate at $t = t_1$:
$$\delta_{i+1} = \delta_i + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$\Delta\omega_{i+1} = \Delta\omega_i + \frac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$$